Name: ________________________ **Class:** ___________________ **Date:** __________ **ID: A**

Good Questions for Calculus Original questions by Maria Terrell, http://www.math.cornell.edu/~GoodQuestions/

True/False

Indicate whether the statement is true or false.

____ 1. 2.1.2 [Q] **True** or **False**.If $f(x) = \dfrac{x^2 - 4}{x - 2}$ and $g(x) = x + 2$ then we can say the functions f and g are equal.

____ 2. 2.2.1. [Q] **True** or **False**. As x increases to 100, $f(x) = \dfrac{1}{x}$ gets closer and closer to 0 so the limit as x goes to 100 of $f(x)$ is 0. Be prepared to justify your answer.

____ 3. 2.2.2 [P] **True** or **False**. $\lim\limits_{x \to a} f(x) = L$ means that if x_1 is closer to a than x_2, is, then $f(x_1)$ will be closer to L than $f(x_2)$ is. Be prepared to justify your answer with an argument or counterexample.

____ 4. 2.2.3 [P] You're trying to guess $\lim\limits_{x \to 0} f(x)$. You plug in $x = 0.1, 0.01, 0.001, \ldots.$ and get $f(x) = 0$ for all these values. In fact, you're told that for all $n = 1, 2, \ldots$, we have $f(1)$, $f(\dfrac{1}{10^n}) = 0$.

True or **False**: Since the sequence 0.1, 0.01, 0.001, . . . goes to 0, we know $\lim\limits_{x \to 0} f(x) = 0$.

____ 5. 2.4.1 [P] **True** or **False**: Let $P(t)$ = "the cost of parking in New York City's parking garages for t hours". So, $P(t) = \$20$ per hour or fraction thereof. For example, if you are in the garage for two hours and one minute, you pay \$60. If t_0 closely approximates some time, T, then $P(t_0)$ closely approximates $P(T)$. Be prepared to justify your answer.

____ 6. 2.4.6 [Q] **True** or **False**. You were once exactly 3 feet tall.

____ 7. 2.4.7 [P] **True** or **False**. At some time since you were born your weight in pounds equaled your height in inches.

____ 8. 2.4.8 [D] **True** or **False**. Along the Equator, there are two diametrically opposite sites that have exactly the same temperature at the same time.

____ 9. 2.4.9 [Q] Suppose that during half-time at a basketball game the score of the home team was 36 points.

True or **False**: There had to be at least one moment in the first half when the home team had exactly 25 points.

____ 10. 2.4.10 [D] **True** or **False**. $x^{100} - 9x^2 + 1$ has a root in $[0, 2]$.

Name: ________________________ **ID: A**

____ 11. 2.4.11 [P] **True** or **False**. The function $f(x)$ defined below is continuous at $x = 0$.

$$f(x) = \begin{cases} x^2 & x \text{ is rational} \\ -x^2 & x \text{ is irrational} \end{cases}$$

____ 12. 2.5.1 [Q] **True** or **False**. Consider a function $f(x)$ with the property that $\lim_{x \to a} f(x) = 0$.

Now consider another function $g(x)$ also defined near a. Then $\lim_{x \to a} \left[f(x)g(x) \right] = 0$.

____ 13. 2.5.2 [Q] **True** or **False**. If $\lim_{x \to a} f(x) = \infty$ and $\lim_{x \to a} g(x) = \infty$, then $\lim_{x \to a} [f(x) - g(x)] = 0$.

____ 14. 2.5.5 [Q] **True** or **False**. A function can cross its horizontal asymptote.

____ 15. 2.7.1 [Q] **True** or **False**. The function $f(x) = x^{1/3}$ is continuous at $x = 0$.

____ 16. 2.7.2 [Q] **True** or **False**. If $f(x) = x^{1/3}$ then $f'(0)$ exists.

____ 17. 2.7.3 [P] If $f(x) = x^{\frac{1}{3}}$ then there is a tangent line at $\left(0,0\right)$.

____ 18. 2.7.4 [P] **True** or **False**. The function $f(x) = |x|$ has a derivative at $x = 0$.

____ 19. 2.7.5 [Q] **True** or **False**. The function $g(x) = x|x|$ has a derivative at $x = 1$.

____ 20. 3.1.2 [D] **True** or **False**. For all n and all x, $\frac{d}{dx}|x|^n = n|x|^{n-1}$.

____ 21. 3.2.4 [Q] The Constant Multiple Rule tells us

$$\frac{d}{dx} cf(x) = c\frac{d}{dx} f(x)$$

and the Product Rule says

$$\frac{d}{dx} cf(x) = c\frac{d}{dx} f(x) + f(x)\frac{d}{dx} c$$

True or **False**. The two rules agree. Be prepared to justify your answer.

____ 22. 3.4.6 [D] We know that $\frac{d}{dx}\left(\sin(x)\right) = \cos(x)$.

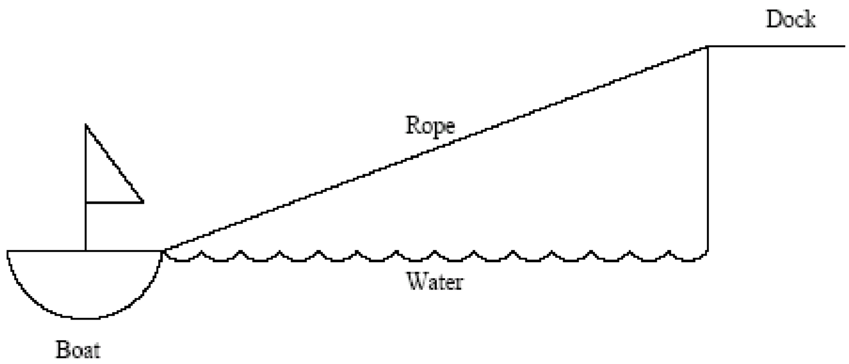
True or **False:** $\frac{d}{dx}\left(\sin(2x)\right) = \cos(2x)$

____ 23. 3.7.1 [Q] **True** or **False**. $\frac{d}{dx}\ln(\pi) = \frac{1}{\pi}$

____ 24. 3.7.3 [P] When you read in the newspaper thing like inflation rate, interest rate, birth rate, etc., it always means $\frac{f'}{f}$, not f' itself.

True or **False**: $\frac{f'}{f}$ is not the derivative of a function.

____ 25. 4.1.3 [P] A boat is drawn close to a dock by pulling in the rope at a constant rate.
True or **False**. The closer the boat gets to the dock, the faster it is moving.



____ 26. 4.2.1 [Q] **True** or **False**. If $f(x)$ is continuous on a closed interval, then it is enough to look at the points where $f'(x) = 0$ in order to find its absolute maxima and minima. Be prepared to justify your answer.

____ 27. 4.3.3 [Q] If $f''(a) = 0$, then f has an inflection point at a.

____ 28. 4.3.6 [Q] **True** or **False**.
For $f(x) = |x|$ on the interval $\left[-\frac{1}{2}, 2\right]$, you can find a point c in $\left(-\frac{1}{2}, 2\right)$ such that

$$f'(c) = \frac{f(2) - f\left(-\frac{1}{2}\right)}{2 - \left(-\frac{1}{2}\right)}$$

____ 29. 4.3.8 [Q] A racer is running back and forth along a straight path. He finishes the race at the place where he began.
True or **False**. There had to be at least one moment, other than the beginning and the end of the race, when he ”stopped” (i.e., his speed was 0). Be prepared to give a proof or counterexample.

____ 30. 4.6.2 [Q] You are given a continuous function, for which $f''(x) > 0$ for all reals, except at $x = a$.
True or **False**. f might have an absolute maximum at $x = a$.
Be prepared to give a counterexample or justify your answer.

____ 31. 4.9.1 [P] Suppose you are told that the acceleration function of an object is a continuous function $a(t)$. Let's say you are given that $v(0) = 1$.
True or False: You can find the position of the object at any time t.

____ 32. 4.9.2 [P] Let $f(x) = \frac{1}{x^2}$, and $F(x)$ be an antiderivative of f with the property $F(1) = 1$.
True or **False**. $F(-1) = 3$.

____ 33. 4.9.4 [Q] **True** or **False**: An antiderivative of a sum of functions, $f + g$, is an antiderivative of f plus an antiderivative of g.

____ 34. 4.9.5 [P] An antiderivative of a product of functions, fg, is an antiderivative of f times an antiderivative of g.

____ 35. 5.1.1 [Q] **True** or **False**. If a piece of string has been chopped into n small pieces and the i^{th} piece is $\triangle x_i$ inches long, then the total length of the string is exactly $\sum_{i=1}^{n} \triangle x_i$

____ 36. 5.2.1 [Q] Let f be a continuous function on the interval $[a,b]$. **True** or **False**:

$$\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

may lead to different limits if we choose the x^* to be the left-endpoints instead of midpoints.

____ 37. 5.2.2 [Q] **True** or **False**. If f is continuous on the interval $[a,b]$, then $\int_a^b f(x)dx$ is a number.

____ 38. 5.3.3 **True** or **False**. If $\int f(x)\,dx = \int g(x)\,dx$, then $f(x) = g(x)$.

____ 39. 5.3.4 [Q] **True** or **False**: If $f'(x) = g'(x)$, then $f(x) = g(x)$

____ 40. 5.4.2 [P] Let f be a continuous function on the interval $[a,b]$. **True** or **False**. There exist two constants m and M, such that $m(b-a) \le \int_a^b f(x)\,dx \le \mathrm{M}(b-a)$

____ 41. 5.4.3 [Q] **True** or **False**. If f is continuous on the interval $[a,b]$, then $\frac{d}{dx}\left(\int_a^b f(x)dx\right) = f(x)$.

Name: ________________________ **ID: A**

Multiple Choice

Identify the choice that best completes the statement or answers the question.

____ 42. 2.1.1 [Q] Let f be the function defined by $f(x) = \sin x + \cos x$ and let g be the function defined by $g(u) = \sin u + \cos u$, for all real numbers x and u. Then,

a. there is not enough information is given to determine if f and g are the same.
b. if x and u are different numbers, f and g are different functions
c. f and g are exactly the same functions

____ 43. 2.1.3 [P] Imagine that there is a rope around the equator of the earth. Add a *20* meter segment of rope to it. The new rope is held in a circular shape centered about the earth. Then the following can walk beneath the rope without touching it:

a. an ant
b. I (the student)
c. an amoeba
d. all of the above

____ 44. 2.1.4 [P] Given two infinite decimals a = .3939393939... and b = .67766777666... , their sum $a + b$

a. can be defined precisely by using successively better approximations.
b. is not a real number because the pattern may not be predictable indefinitely.
c. is not a number because not all infinite decimals are real numbers.
d. is not defined because the sum of a rational and irrational number is not defined.

____ 45. 2.2.4 [P] Suppose you have an infinite sequence of closed intervals, each one contains the next, and suppose too that the width of the nth interval is less than $\frac{1}{n}$. If a and b are in each of these intervals, then

a. either a or b must be an endpoint of one of the intervals
b. $a = b$
c. a and b are very close but they don’t have to be equal

Name: ________________________ **ID: A**

____ 46. 2.2.5 [D] Consider the function $f(x) = \begin{cases} x^2 & x \text{ is rational}, x \neq 0 \\ -x^2 & x \text{ is irrational} \\ \text{undefined} & x = 0 \end{cases}$

Then

a. $\lim_{x \to a} f(x)$ exists for infinitely many a
b. there is no a for which $\lim_{x \to a} f(x)$ exists
c. there may be some a for which $\lim_{x \to a} f(x)$ exists, but it is impossible to say without more information
d. $\lim_{x \to a} f(x)$ exists only when $a = 0$

____ 47. 2.3.1 [Q] The statement "Whether or not $\lim_{x \to a} f(x)$ exists, depends on how $f(a)$ is defined." is true

a. sometimes
b. always
c. never

____ 48. 2.3.2 [Q] If a function f is not defined at $x = a$,

a. $\lim_{x \to a} f(x)$ could be 0
b. $\lim_{x \to a} f(x)$ must approach 1
c. $\lim_{x \to a} f(x)$ cannot exist
d. none of the above.

____ 49. 2.3.3 [Q] If $\lim_{x \to a} f(x) = 0$ and $\lim_{x \to a} g(x) = 0$, then $\lim_{x \to a} \frac{f(x)}{g(x)}$

a. must exist
b. not enough information
c. does not exist

____ 50. 2.3.4 [D] The reason that $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist is:

a. because 1/0 is undefined
b. because no matter how close x gets to 0, there are x's near 0 for which $\sin(\frac{1}{x}) = 1$, and some for which $\sin(\frac{1}{x}) = -1$.
c. because the function values oscillate around 0
d. all of the other answers are true

Name: ________________________ **ID: A**

____ 51. 2.3.5 [D] $\lim_{x \to 0} x^2 \sin(\frac{1}{x})$

a. because no matter how close x gets to 0, there are x's near 0 for which $\sin(\frac{1}{x}) = 1$, and some for which $\sin(\frac{1}{x}) = -1$.

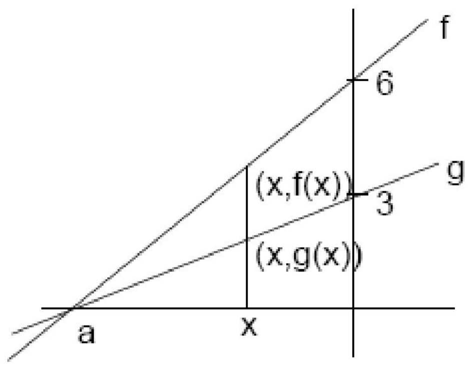
b. does not exist because 1/0 is undefined

c. does not exist because the function values oscillate around 0

d. equals 1

e. equals 0

____ 52. 2.3.6 [D] Suppose you have two linear functions f and g shown below.



Then $\lim_{x \to a} \frac{f(x)}{g(x)}$ is

a. does not exist

b. not enough information

c. 2

d. 3

____ 53. 2.4.2 [Q] A drippy faucet adds one milliliter to the volume of water in a tub at precisely one second intervals. Let f be the function that represents the volume of water in the tub at time t.

a. f is continuous for all t other than the precise instants when the water drips into the tub

b. f is a continuous function at every time t

c. not enough information to know where f is continuous.

d. f is not continuous at any time t

Name: ________________________ **ID: A**

____ 54. 2.4.3 [P] A drippy faucet adds one milliliter to the volume of water in a tub at precisely one second intervals. Let g be the function that represents the volume of water in the tub as a function of the depth of the water, x, in the tub.

a. g is a continuous function at every depth x
b. not enough information to know where g is continuous
c. g is not continuous at any depth x
d. there are some values of x at which g is not continuous

____ 55. 2.4.4 [Q] You know the following statement is true:

"If $f(x)$ is a polynomial, then $f(x)$ is continuous."

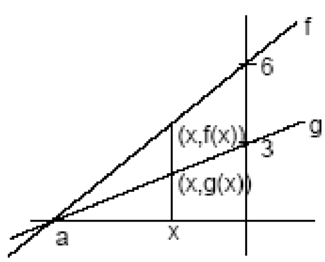
Which of the following is also true?

a. If $f(x)$ is not a polynomial, then it is not continuous.
b. If $f(x)$ is continuous, then it is a polynomial.
c. If $f(x)$ is not continuous, then it is not a polynomial.

____ 56. 2.4.5 [D] You decide to estimate e^2 by squaring longer decimal approximations of $e = 2.71828\ldots$

a. This is a good idea because $y = e^x$ is a continuous function.
b. This is a good idea because $y = x^2$ is a continuous function.
c. This is a bad idea because e is irrational.
d. This is a good idea because e is a rational number.

____ 57. 2.5.3 [P] Suppose you have two linear function f and g shown below.



Then $\lim_{x \to \infty} \frac{f(x)}{g(x)}$ is:

a. 2
b. 3
c. does not exist
d. not enough information

Name: ________________________ **ID: A**

____ 58. 2.5.4 [Q] What is the maximum number of horizontal asymptotes that a function can have?

a. one
b. two
c. as many as we want
d. three

____ 59. 2.6.1 [P] At $(0,0)$ the graph of $f(x) = |x|$

a. has infinitely many tangent lines
b. has a tangent line at $y = 0$
c. has two tangent lines $y = -x$ and $y = x$
d. has no tangent line

____ 60. 2.6.2 [P] The line tangent to the graph of $f(x) = x$ at $(0,0)$

a. does not exist
b. is $y = x$
c. is not unique. There are infinitely many tangent lines.
d. is $y = 0$

____ 61. 2.6.3 [Q] If the radius of a circle increases from r_1 to r_2, then the average rate of change of the area of the circle is

a. equal to $2\pi \dfrac{r_1 + r_2}{2}$
b. all of the above
c. greater than $2\pi r_1$
d. less than $2\pi r_2$

____ 62. 2.6.4 [P] Consider the function

$$f(x) = \begin{cases} x^2 & x \text{ is rational} \\ -x^2 & x \text{ is irrational} \end{cases}$$

Does $f'(0)$ exist?

a. no
b. yes
c. not possible to tell

____ 63. 2.6.5 [P] Water is being poured into a cylindrical vase. The height of the water changes as more water is poured in. The instantaneous change in the height with respect to the volume of water in the vase

a. varies inversely as the cube of the radius
b. not enough information to tell.
c. is constant

____ 64. 2.7.6 [P] The derivative of $f(x) = x|x|$ at $x = 0$

a. is 0
b. does not exist, because the left and right hand limits do not agree
c. does not exist, because $|x|$ is not differentiable at $x = 0$
d. does not exist, because f is defined piecewise

____ 65. 2.8.1 [P] If $f'(a)$ exists, $\lim_{x \to a} f(x)$

a. equals $f'(a)$
b. equals $f(a)$
c. it must exist, but there is not enough information to determine it exactly
d. it may not exist

____ 66. 2.8.2 [P] Your mother says "If you eat your dinner, you can have dessert". You know this means, "If you don't eat your dinner, you cannot have dessert". Your calculus teacher says, "If f if differentiable at x, f is continuous at x". You know this means

a. if f is not differentiable at x, f is not continuous at x.
b. if f is not continuous at x, f is not differentiable at x.
c. knowing f is not continuous at x, does not give us enough information to deduce anything about whether the derivative of f exists at x.

____ 67. 2.8.3. [Q] A slow freight train chugs along a straight track. The distance it has traveled after x hours is given by a function $f(x)$ An engineer is walking along the top of the box cars at the rate of 3 mi/hr in the same direction as the train is moving. The speed of the man relative to the ground is

a. $f'(x) - 3$
b. $f(x) + 3$
c. $f(x) - 3$
d. $f'(x) + 3$

____ 68. 2.8.4 [Q] Suppose $f'(x)$ exists for all x in (a, b). Read the following four statements:

(I) $f(x)$ is continuous on (a, b)
(II) $f(x)$ is continuous at $x = a$
(III) $f(x)$ is defined for all x in (a, b)
(IV) $f'(x)$ is differentiable on (a, b)

a. None of I, II, III and IV
b. I, I and I
c. All of I, II, III and IV
d. Only I
e. I and I

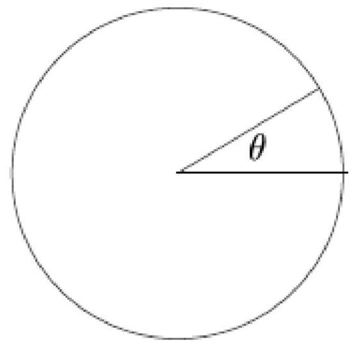
____ 69. 3.1.1 [Q] $\frac{d}{dx}\left(e^7\right)$ equals

a. 0
b. e^7
c. $7e^6$

____ 70. 3.1.3 [P] $\lim_{x \to 1} \frac{x^{10} - 1}{x - 1}$

a. does not exist, because $\frac{0}{0}$ is not defined
b. equals 10, because it is exactly the derivative of x^{10}, at $x = 1$
c. equals 1, because $(1)^9 = 1$.

____ 71. 3.1.4 [Q] Suppose you cut a slice of pizza from a circular pizza of radius r, as shown.



As you change the size of the angle θ, you change the area of the slice, $A = \frac{1}{2}r^2\theta$. Then A' is

a. $\frac{1}{2}r^2$
b. not possible to determine from the given information
c. $r\theta$

____ 72. 3.1.5 [Q] The radius of a snowball changes as the snow melts. The instantaneous change in radius with respect to volume is

a. $\frac{dV}{dr} + \frac{dr}{dV}$
b. cannot be determined
c. $\frac{dV}{dr}$
d. $\frac{dr}{dV}$

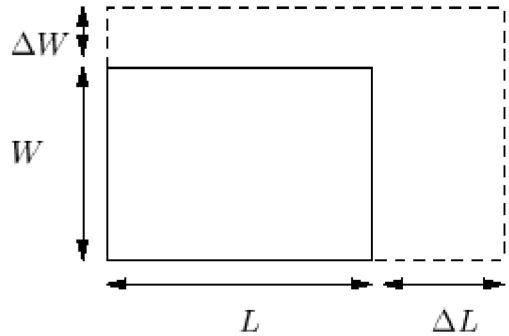
____ 73. 3.1.6 [P] Gravel is poured into a canonical pile. The rate at which gravel is added to the pile is

a. $\frac{dr}{dt}$
b. $\frac{dV}{dr}$
c. $\frac{dV}{dt}$
d. none of the above

____ 74. 3.2.1 [Q] We know $f(1) = 1$ and $f_0(1) = 3$. Then $\frac{d}{dx}\frac{f(x)}{x^2}\bigg|_{x=1}$ equals

a. 3/2
b. 1
c. -1

____ 75. 3.2.2 [P] Suppose that over an interval of time, $\triangle t$, the length, L, and width, W, of a rectangle grow to $L+\triangle L$ and $W+\triangle W$ respectively as in the sketch below.



The average rate of change in A over the interval of time, t is:

a. $\dfrac{LW}{\Delta t}+\dfrac{(\Delta L)(\Delta W)}{\Delta t}$

b. $L\dfrac{\Delta W}{\Delta t}+W\dfrac{\Delta L}{\Delta t}+\dfrac{(\Delta W)(\Delta L)}{\Delta t}$

c. $\dfrac{LW}{\Delta t}+L\dfrac{\Delta W}{\Delta t}+W\dfrac{\Delta L}{\Delta t}+\dfrac{(\Delta L)(\Delta W)}{\Delta t}$

d. $L\dfrac{\Delta W}{\Delta t}+W\dfrac{\Delta L}{\Delta t}$

____ 76. 3.2.3 [P] In the problem above, assume that L and W are differentiable functions of time, (i.e.$\dfrac{dL}{dt}$ and $\dfrac{dW}{dt}$ both exist for all t). The reason $\dfrac{dA}{dt}$ is the sum of the limits of the first two terms, and that the third term "disappears" when you take the limit is:

a. the terms in the numerator are both approaching 0, and therefore their product goes to 0 faster than the term in the denominator does
b. it is not possible to tell why the third term approaches 0.
c. you can cancel one of the close to zero terms in the numerator with a close to zero term in the denominator, so the limit of the remaining term is 0
d. $\dfrac{dL}{dt}$ exists for each t and is some number, hence the limit of the third term is 0.

____ 77. 3.4.1 [P] $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$ means that

a. $\sin x = x$ for x near 0.
b. $\dfrac{0}{0}=1$
c. you can cancel the x's
d. the tangent to the graph of $y=\sin x$ at $(0,0)$ is the line $y=x$

____ 78. 3.4.2 [Q] If $f(x) = \sin x$ then

a. $f(x) = f''''(x)$
b. $f'(x) = \cos(x)$
c. $f(x) = -f''(x)$
d. all of the above.

____ 79. 3.4.3 [Q] If $f(x) = \sin x \cos x$, $f'(x) =$

a. $\cos 2x$
b. $1 - 2\sin^2(x)$
c. $2\cos^2(x)$
d. none of the above
e. all of the above

____ 80. 3.4.4 [Q] If $f(x) = \tan x$, $f'(x) =$

a. $\cot x$
b. $-\cot x$
c. $1 + \tan^2(x)$
d. all of the above
e. none of the above

____ 81. 3.4.5 [P] $\lim\limits_{h \to 0} \dfrac{\sin(2x+h) - \sin(2x)}{h}$ equals

a. does not exist because 0/0 is not defined
b. $\cos x$
c. zero
d. $\cos(2x)$

____ 82. 3.5.1 [P] $\dfrac{d}{dr}(\sin x + e^{\sin x})$ equals

a. not enough information
b. $\cos x + e^{\sin x} \cos x$
c. $\cos x + e^{\cos x}$
d. $\cos r + e^{\sin r} \cos r$

____ 83. 3.5.2 [P] Suppose a deli clerk can slice a stick of pepperoni (assume the tapered ends have been removed) by hand at the rate of 2 inches per minute, while a machine can slice pepperoni at the rate of 10 inches per minute. Then $\dfrac{dV}{dt}$ for the machine is 5 times greater than $\dfrac{dV}{dt}$ for the deli clerk. This is explained by the

a. product rule
b. addition rule
c. chain rule
d. quotient rule

____ 84. 3.5.3. [Q] If f and g are both differentiable and $h = f \circ g$, $h'(2)$ equals (d)

a. $f'(2) \circ g'(2)$
b. $f'(2)g'(2)$
c. $f'(g(2))g'(2)$
d. $f'(g(x))g'(2)$

____ 85. 3.5.4 [P] The area of a circle, $A = \pi r^2$, changes as its radius changes. If the radius changes with respect to time, the change in area with respect to time is

a. Not enough information
b. $\dfrac{dA}{dt} = 2\pi r \dfrac{dr}{dt}$
c. $\dfrac{dA}{dr} = 2\pi r$
d. $\dfrac{dA}{dr} = 2\pi r + \dfrac{dr}{dt}$

____ 86. 3.6.1 [D] When you solve for y' in an implicit differentiation problem, you have to solve a quadratic equation.

a. sometimes
b. never
c. always

____ 87. 3.6.2 [Q] If $g(x) = \sin^{-1} x$, then $g'(x)$ is

a. $\csc x \cot x$
b. $\dfrac{1}{\sqrt{1-x^2}}$
c. $\dfrac{1}{\cos x}$
d. $-\dfrac{\cos x}{\sin^2 x}$

____ 88. 3.6.3 [P] The slope of the line tangent to the graph of $x = \sin y$ at the point $(0, \pi)$ is

a. -1
b. 1
c. not defined

____ 89. 3.7.2 [Q] Your calculus book says that $e = \lim\limits_{n \to \infty} \left(1 + \dfrac{1}{n}\right)^n$. This means:

a. e is not really a number because it is a limit
b. the sequence of numbers $\left(\dfrac{2}{1}\right), \left(\dfrac{3}{2}\right)^2, \left(\dfrac{4}{3}\right)^3, \ldots., \left(\dfrac{101}{100}\right)^{100}$,..... get as close as you want to the number e
c. e cannot be computed

____ 90. 2.9 & 3.8.1 [P] If $e^{.5}$ is approximated by using the tangent line to the graph of $f(x) = e^x$ at $(0, 1)$, and we know $f_0(0) = 1$, the approximation is

a. $1 + e^{.5}$
b. .5
c. 1 + .5

____ 91. 2.9 & 3.8.2 [Q] The line tangent to the graph of $f(x) = \sin x$ at $\left(0, 0\right)$ is $y = x$. This implies that

a. The line $y = x$ touches the graph of $f(x) = \sin x$ at exactly one point, $\left(0, 0\right)$
b. $y = x$ is the best straight line approximation to the graph of f for all x
c. $\sin(.0005) \approx .0005$

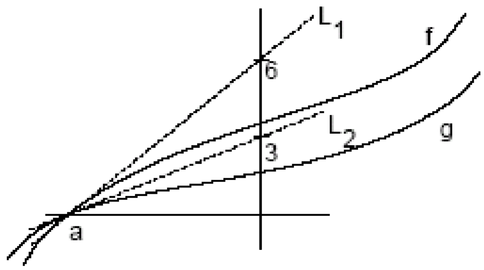
____ 92. 2.9 & 3.8.3 [P] The line $y = l$ is tangent to the graph of $f(x) = \cos x$ at $\left(0, 1\right)$. This means that

a. tangent lines can intersect the graph of f infinitely many times
b. the only x-values for which $y = l$ is a good estimate for $y = \cos x$ are those that are close enough to 0
c. the farther x is from 0, the worse the linear approximation is

____ 93. 2.9 & 3.8.4 [P] The error of using the tangent line $y = f(a) + f'(a)(x - a)$ to approximate $y = f(x)$ is $\mathrm{E}(x) = f(x) - \left[\, f(a) + f'(a)(x - a) \,\right]$. Then $\lim\limits_{x \to a} \dfrac{\mathrm{E}(x)}{(x - a)}$

a. must be 0
b. depends on the value of a
c. might not exist

____ 94. 2.9 & 3.8.5 [D] Suppose you have two functions f and g shown below, and their tangent lines L_1 and L_2.



The $\lim_{x \to a} \dfrac{f(x)}{g(x)}$.

a. does not exist, because $\dfrac{0}{0}$ does not exist
b. cannot be determined with this information. You need to know a
c. is 3
d. is 2

____ 95. 2.9 & 3.8.6 [Q] Suppose that $f''(x) < 0$ for x near a point a. Then the linearization of f at a is

a. an over approximation
b. an under approximation
c. unknown without more information.

____ 96. 2.9 & 3.8.7 [Q] Peeling an orange changes its volume V. What does ΔV represent?

a. the volume of the ”edible part” of the orange
b. the volume of the rind
c. the surface area of the orange
d. -1 × (the volume of the rind)

____ 97. 2.9 & 3.8.8 [P] If you divide the volume of the rind of a thin-skinned orange by the thickness of the rind you get a good estimate for

(I) The surface area of the original orange
(II) The surface area of the edible part
(III) The change in the volume of the orange
(IV) The square of the radius of the orange

a. IV
b. III
c. I and II
d. I only

____ 98. 2.9 & 3.8.9 [P] Imagine that you increase the dimensions of a square with side x_1 to a square with side length x_2. The change in the area of the square, $\triangle A$, is approximated by the differential *dA*. In this example, *dA* is

a. $x_2^2 - x_1^2$
b. $\left(x_2 - x_1\right)^2$
c. $\left(x_2 - x_1\right)2x_1$
d. $\left(x_2 - x_1\right)2x_2$

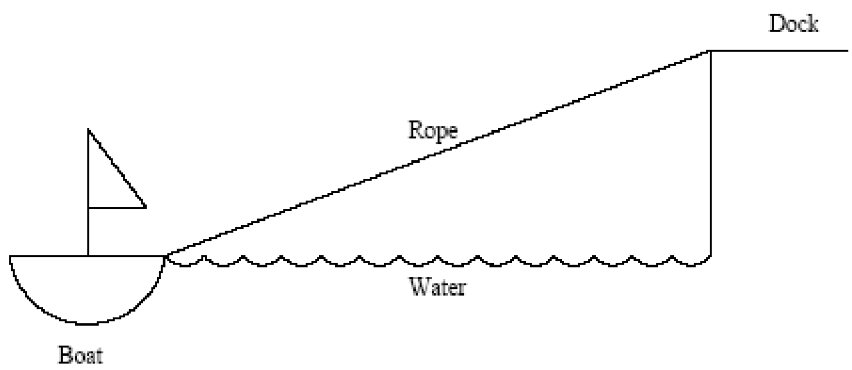
____ 99. 2.9 & 3.8.10 [Q] Imagine that you increase the dimensions of a square with side x_1 to a square with side length x_2. If you estimate the change in the area of the square, ΔA by the differential $dA = 2x_1\left(x_2 - x_1\right)$, this will result in an

a. exactly equal
b. underestimate
c. overestimate

____ 100. 4.1.1 [Q] As gravel is being poured into a conical pile, its volume V changes with time. As a result, the height h and radius r also change with time. Knowing that at any moment $V = \dfrac{1}{3}\pi r^2 h$, the relationship between the changes with respect to time in the volume, radius and height is

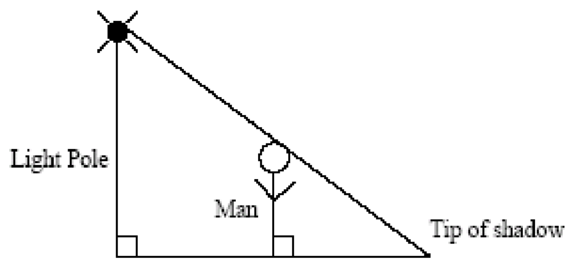
a. $\dfrac{dV}{dt} = \dfrac{1}{3}\pi\left(2r\dfrac{dr}{dt}h + r^2\dfrac{dh}{dt}\right)$
b. $\dfrac{dV}{dt} = \dfrac{1}{3}\pi\left(2r\dfrac{dr}{dt} \cdot \dfrac{dh}{dt}\right)$
c. $\dfrac{dV}{dt} = \dfrac{1}{3}\pi\left((r^2)(1) + 2r\dfrac{dh}{dt}h\right)$
d. $\dfrac{dV}{dt} = \dfrac{1}{3}\pi\left(2rh + r^2\dfrac{dh}{dt}\right)$

____ 101. 4.1.2 [P] A boat is drawn close to a dock by pulling in a rope as shown. How is the rate at which the rope is pulled in related to the rate at which the boat approaches the dock?



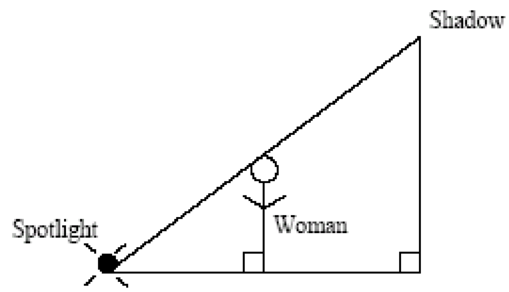
a. It depends on how close the boat is to the dock.
b. They are equal.
c. One is a constant multiple of the other.

____ 102. 4.1.4 [P] A streetlight is mounted at the top of a pole. A man walks away from the pole. How is the rate at which he walks away from the pole related to the rate at which his shadow grows?



a. It depends also on how close the man is to the pole.
b. They are equal.
c. One is a constant multiple of the other.

____ 103. 4.1.5 [P] A spotlight installed in the ground shines on a wall. A woman stands between the light and the wall casting a shadow on the wall. How are the rate at which she walks away from the light and rate at which her shadow grows related?

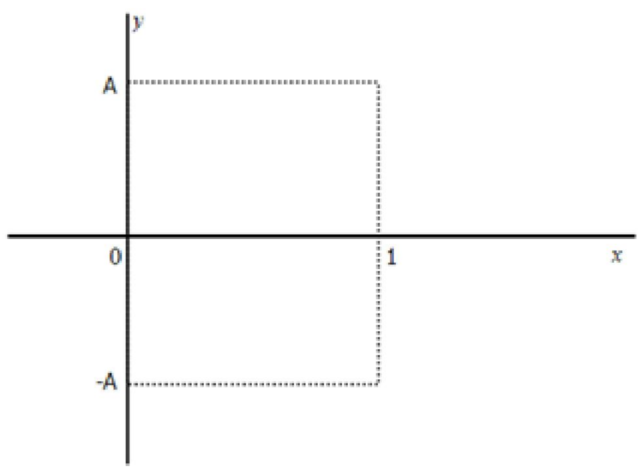


a. One is a constant multiple of the other.
b. It depends also on how close the woman is to the pole.
c. They are equal.

____ 104. 4.2.2 [P] Let $f(x)$ be a differentiable function on a closed interval with $x = a$ being one of the endpoints of the interval. If $f'(a) > 0$ then,

a. f cannot have an absolute maximum at $x = a$.
b. f could have either an absolute maximum or an absolute minimum at $x = a$.
c. f must have an absolute minimum at $x = a$.

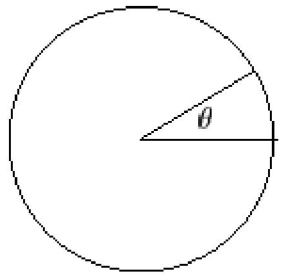
____ 105. 4.2.3 [P] Let f be a continuous function on the closed interval $0 \le x \le 1$. There exists a positive number A so that the graph of f can be drawn inside the rectangle $0 \le x \le 1$, $-A \le x \le A$



The above statement is:

a. Not enough information.
b. Always true.
c. Sometimes true.

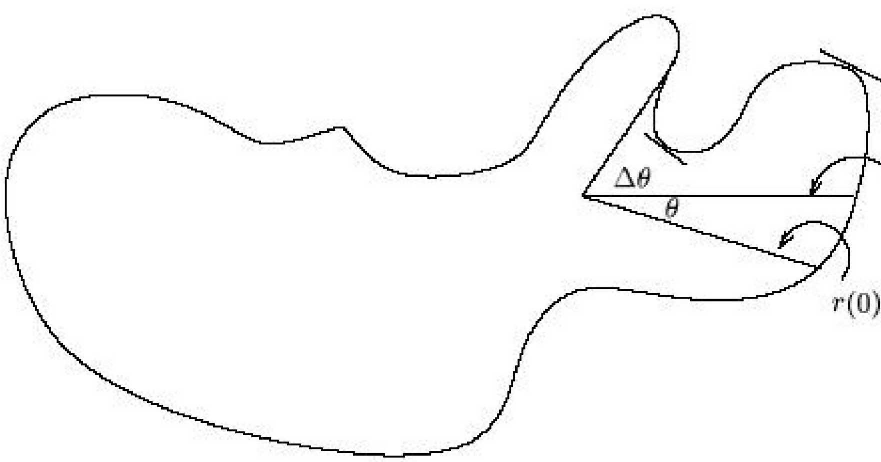
____ 106. 4.2.4 [Q] Suppose you cut a slice of pizza from a circular pizza of radius r, as shown.



As you change the size of the angle θ, you change the area of the slice, $A = \frac{1}{2} r^2 \theta$. Then $A' =$

a. $r \frac{dr}{d\theta} \theta + \frac{1}{2} r^2$, and $\frac{dr}{d\theta}$ is not 0
b. $r \frac{d\theta}{dr}$
c. $r\theta + \frac{1}{2} r^2 \frac{d\theta}{dr}$
d. $\frac{1}{2} r^2$

____ 107. 4.2.5 [D] Suppose you cut a slice of pizza from an amoeba shaped pizza, as shown.



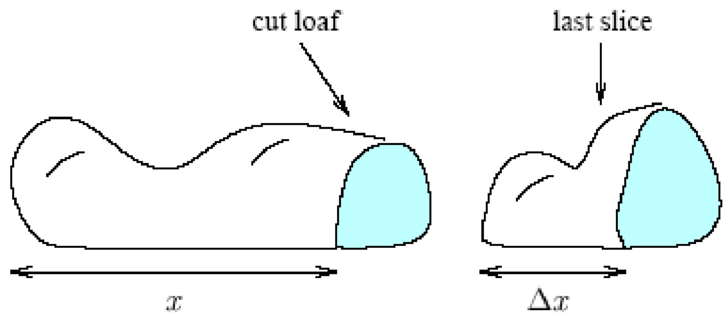
As you change the size of the angle θ, you change the area of the slice. Then $A'(\theta)$ is

a. $\frac{1}{2}(r(\theta))^2$
b. Not enough information. You need an explicit function for the area
c. $\frac{1}{2}(r(\theta))^2 + r(\theta)\frac{dr}{d\theta}\theta$

____ 108. 4.2.6 [D] As a tree grows, its trunk increases its volume by adding a growth ring, around the outside of the trunk (assume the thickness of the growth ring is the same over the whole trunk). The instantaneous rate of change in the volume of the trunk with respect to thickness of the growth ring is

a. the surface area of the trunk.
b. the thickness of the growth ring times the surface area of the trunk.
c. the circumference of the trunk.

____ 109. 4.2.7 [D] When you slice a loaf of bread, you change its volume. Let x be the length of the loaf from one end to the place where you cut off the last slice. Let $V(x)$ be the volume of the loaf of length x (see figure). For each x, $\dfrac{dV}{dx}$ is



a. the area of the cut surface of the loaf where the last slice was removed
b. the volume of the last slice divided by the thickness of the slice.
c. the volume of a slice of bread

____ 110. 4.3.1 [P] An article in the Wall Street Journal's "Heard on the Street" column (Money and Investment August 1, 2001) reported that investors often look at the "change in the rate of change" to help them "get into the market before any big rallies." Your stock broker alerts you that the rate of change in a stock's price is increasing. As a result you

a. can conclude the stock's price is increasing
b. cannot determine whether the stock's price is increasing or decreasing.
c. can conclude the stock's price is decreasing

____ 111. 4.3.2 [P] Imagine that you are skydiving. The graph of your speed as a function of time from the time you jumped out of the plane to the time you achieved terminal velocity is

a. increasing and concave up
b. decreasing and concave up
c. increasing and concave down
d. decreasing and concave down

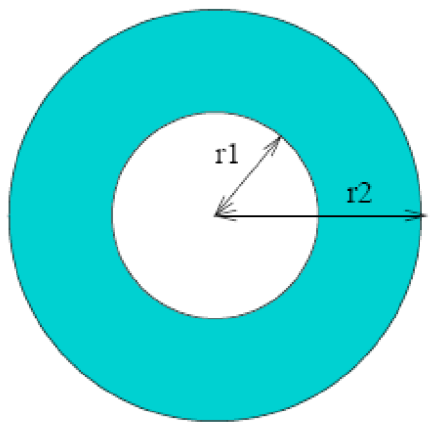
____ 112. 4.3.4 [D] Water is being poured into a "Dixie cup" (a standard cup that is smaller at the bottom than at the top). The height of the water in the cup is a function of the volume of water in the cup. The graph of this function is

a. increasing and concave down
b. increasing and concave up
c. a straight line with positive slope

____ 113. 4.3.5 [P] On a toll road a driver takes a time stamped toll-card from the starting booth and drives directly to the end of the toll section. After paying the required toll, the driver is surprised to receive a speeding ticket along with the toll receipt. Which of the following best describes the situation?

a. The booth attendant does not have enough information to prove that the driver was speeding.
b. The booth attendant can prove that the driver was speeding during his trip.
c. The driver will get a ticket for a lower speed than his actual maximum speed.
d. Both (*b*) and (*c*)

____ 114. 4.3.7 [P] The region between two concentric circles of radius r_1 and r_2 is called an annulus. If $r_2 > r_1$, the area of the annulus is $\pi\left(r_2^2 - r_1^2\right)$.



a. There must be a radius, r, between r_1 and r_2 for which the rectangle with base $r_2 - r_1$ and height $2\pi r$ is exactly equal to the area of the annulus.

b. This area cannot be approximated by the area of rectangles because the circles are concentric.

c. This area can be approximated by a sum of areas of rectangles, but there is no single rectangle that has exactly the same area.

____ 115. 4.3.9 [D] Two racers start a race at the same moment and finish in a tie. Which of the following must be true?

a. The racers' speeds at the end of the race must have been exactly the same.
b. The racers had to have the same speed at some moment, but not necessarily at exactly the same time.
c. At some point during the race the two racers were not tied.
d. The racers must have had the same speed at exactly the same time at some point in the race.

Name: ________________________ **ID: A**

____ 116. 4.3.10 [D] A solid cone with a circular base is sliced parallel to its base. The top and bottom of each slice are circles of radius r_1 and r_2, say $r_2 > r_1$.

a. The volume of the slice cannot be approximated by the volume of a cylinder because the circles do not have the same radius.
b. The volume of this slice can be approximated by the volume of a cylinder with the same thickness as the slice, but there is not necessarily a cylinder that has exactly the same volume as the slice.
c. There must be a radius, r, between r_1 and r_2 for which the volume of the cylinder of radius, r, with height equal to the thickness of the slice is exactly equal to the volume of the slice.

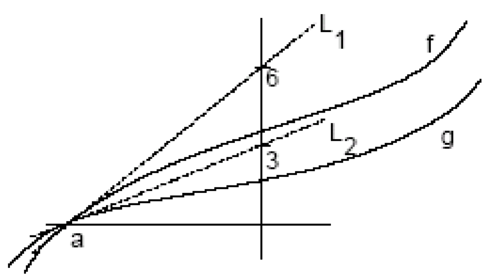
____ 117. 4.5.1 [Q] Consider the functions $f(x) = e^x$ and $g(x) = x^{1,000,000}$. As $x \to \infty$ which of the following is true?

a. They grow at the same rate like all exponentials.
b. We cannot determine.
c. g grows faster than f
d. f grows faster than g.

____ 118. 4.5.2 [Q] The limit $\lim\limits_{x \to \infty} \left[xe^{1/x} - x \right]$

a. Is 1 because $xe^{1/x}$ grows faster than x.
b. Converges to 0.
c. Does not exist because $\infty - \infty$ is not defined.
d. Converges to 1.

____ 119. 4.5.3 [Q] Suppose you have two functions f and g, with linear approximations L_1 and L_2 at $x = a$ as shown below.



a. Does not exist
b. Not enough information
c. 3
d. 2

____ 120. 4.6.1 [Q] A designer wants to introduce a new line of bookcases: he wants to make at least 100 bookcases, but not more than 2000 of them. He predicts the cost of producing x bookcases is $C(x)$. Assume that $C(x)$ is a differentiable function. Which of the following must he do to find the minimum average cos$t$, $c(x) = \dfrac{C(x)}{x}$?

(I) find the points where $c'(x) = 0$ and evaluate $c(x)$ there
(II) compute $c''(x)$ to check which of the critical points in (I) are local maxima.
(III) check the values of c at the endpoints of its domain.

a. *I* only
b. *I* and *III* only
c. *I*, *II* and *III*
d. *I* and *II* only

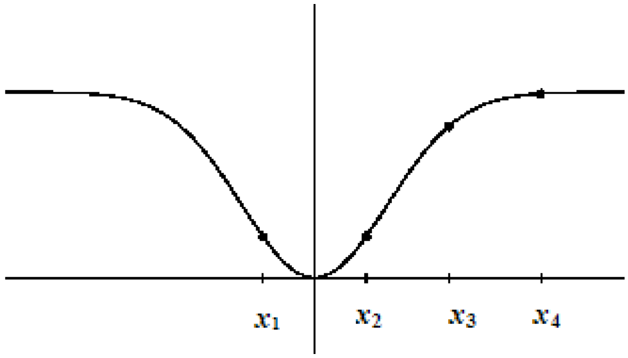
____ 121. 4.6.3 [Q] If f is continuous on $\left[a,b\right]$, then

a. there must be local extreme values, but there may or may not be an absolute maximum or absolute minimum value for the function.
b. there must be numbers m and M such that $m \le f(x) \le M$, for $x \in \left[a,b\right]$
c. any absolute max or min would be at either the endpoints of the interval, or atplaces in the domain where $f'(x) = 0$

____ 122. 4.6.4 [D] You have a piece of wire of length L from which you construct a circle and/or a square in such a way that the total enclosed area is maximal. Then

a. you should construct both the square and the circle, but the perimeter of the square must be larger than the perimeter of the circle
b. you should construct both the square and the circle, but the perimeter of the square must be less than the perimeter of the circle
c. you should construct only the square
d. you should construct only the circle

____ 123. 4.8.1 [Q] We will use each of the x_n below as the starting point for Newton's method. For which of them do you expect Newton's method to work and lead to the root of the function?



a. x_1 and x_2 only.
b. x_1, x_2 and x_3 only.
c. x_2 only.
d. All four

____ 124. 4.8.2 [Q] Let f be a differentiable function defined for all x. Starting Newton's method at a point c where $f'(c) = 0$ is:

a. It could work if we are lucky.
b. A good idea, because $x = c$ is a critical point so Newton's method will lead us straight to the root.
c. Is usually a bad idea because we might get stuck.
d. Both (b) and (c).

____ 125. 4.8.3 [Q] Newton's method is a cool technique, because:

a. Both (a) and (b).
b. It can be used to find a solution to $x^7 = 3x^3 + 1$
c. It can help us get decimal representations of numbers like $\sqrt[4]{3}, \sqrt[8]{5}$ and $\sqrt[5]{13}$

____ 126. 4.9.3 [Q] If f is an antiderivative of g, and g is an antiderivative of h, then

a. h is an antiderivative of f
b. h is the second derivative of f
c. h is the derivative of f''

____ 127. 5.1.2 [P] You want to estimate the area underneath the graph of a positive function by using four rectangles of equal width. The rectangles that must give the best estimate of this area are those with height obtained from the:

a. Left endpoints
b. Midpoints
c. Right endpoints
d. Not enough information

____ 128. 5.1.3 [P] Suppose you are slicing an 11 inch long carrot REALLY thin from the greens end to the tip of the root. If each slice has a circular cross section $f(x) = \pi(r(x))^2$ for each x between 0 and 11, and we make our cuts at $x_1, x_2, x_3, \ldots, x_n$, then a good approximation for the volume of the carrot is

a. $\sum_{i=1}^{n} [f(x_{i+1}) - f(x_i)]x_i$
b. $\sum_{i=1}^{n} f(x_i)[x_{i+1} - x_i]$
c. $\sum_{i=1}^{n} f(x_i)x_i$

____ 129. 5.2.3 [P] Read the following four statements and choose the correct answer below. If f is continuous on the interval $[a, b]$, then:

(i) $\int_a^b f(x)\,dx$ is the area bounded by the graph of f, the x-axis and the lines $x = a$ and $x = b$

(ii) $\int_a^b f(x)\,dx$ is a number

(iii) $\int_a^b f(x)\,dx$ is an antiderivative of $f(x)$

(iv) $\int_a^b f(x)\,dx$ may not exist

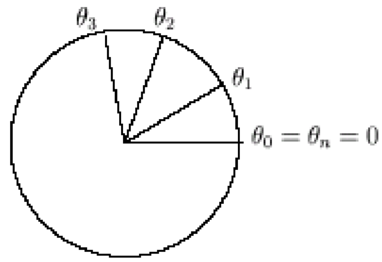
a. (i) and (ii) only
b. (ii) only
c. (i) and (iii) only
d. (iv) only

____ 130. 5.2.4 [P] Water is pouring out of a pipe at the rate of $f(t)$ gallons/minute. You collect the water that flows from the pipe between $t = 2$ and $t = 4$. The amount of water you collect can be represented by:

a. the average of $f(4)$ and $f(2)$ times the amount of time that elapsed
b. $(4-2)f(4)$
c. $f(4) - f(2)$
d. $\int_2^4 f(x)dx$

Name: ________________________ **ID: A**

____ 131. 5.2.5 [D] We cut a circular disk of radius *r* into *n* circular sectors, as shown in the figure, by marking the angles θ_i at which we make the cuts ($\theta_0 = \theta_n$ can be considered to be the angle 0). A circular sector between two angles θ_i and θ_{i+1} has area $\frac{1}{2} r^2 \Delta\theta$ where $\Delta\theta = \theta_{i+1} - \theta_i$



We let $A_n = \sum_{t-0}^{n-1} \frac{1}{2} r^2 \Delta\theta$. Then the area of the disk, A, is given by:

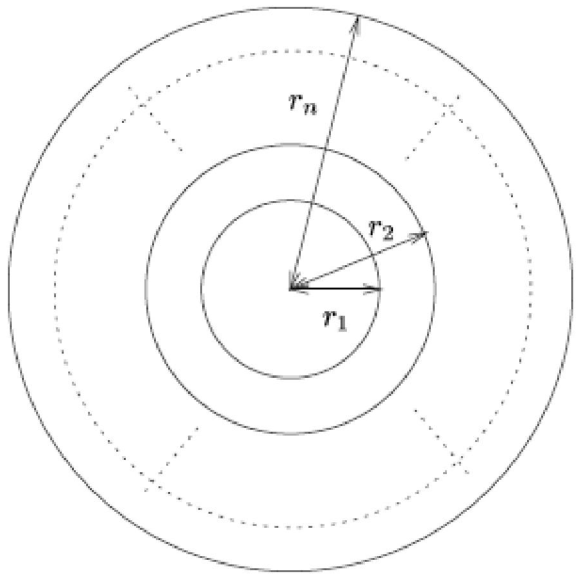
a. $\int_0^{2\pi} \frac{1}{2} r^2 d\theta$

b. $\lim_{n \to \infty} A_n$

c. A_n, independent of how many sectors we cut the disk into.

d. all of the above

____ 132. 5.2.6 [P] Suppose we cut a disk of radius R using n concentric circles, each one of radius r_i (see the figure below and let $r_0 = 0$). We have seen in the MVT problem, that between two radii r_i and r_{i+1}, there exists *a* radius $\overline{r_i}$ such that the area of the annulus between r_i and r_{i+1} is exactly $2\pi\overline{r_i}\left(r_{i+1} - r_i\right)$. Letting $A_n = \sum_{i=0}^{n-1} 2\pi r_i\left(r_{i+1} - r_i\right)$, then the area of the disk, A, equals:



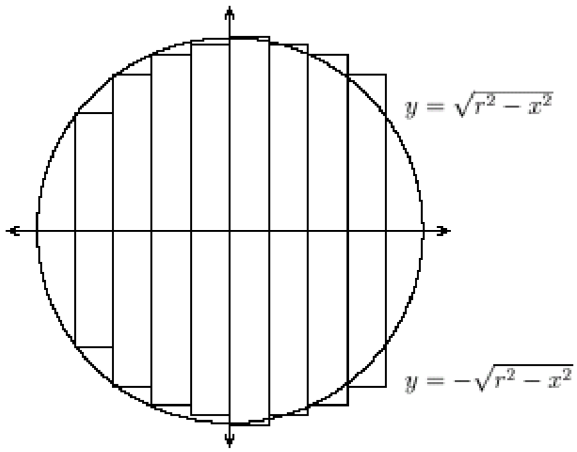
a. $\lim_{n \to \infty} A_n$

b. all of the above

c. $\int_0^R 2\pi r dr$

d. A_n, for all n

____ 133. 5.2.7 [P] Suppose we are going to consider the disk of radius r as the region bounded between the graphs of the functions $\sqrt{r^2 - x^2}$, and $-\sqrt{r^2 - x^2}$. Which of the following statements is true?



a. The area of the disk can be written as a the limit of Riemann Sums of rectangles of length Δx and height $2\sqrt{r^2 - x_i^2}$ where the x_i are a partition of the interval $[-r, r]$.

b. The area of the region is given by the formula: $\int_{-r}^{r} 2\sqrt{r^2 + x^2}\, dx$

c. Both (a) and (b).

d. The area cannot be found this way, because we cannot integrate the function $\sqrt{r^2 - x^2}$.

____ 134. 5.3.1 [Q] A sprinter practices by running various distances back and forth in a straight line in a gym. Her velocity at t seconds is given by the function $v(t)$.

What does $\int_0^{60} |v(t)|dt$ represent?

a. The total distance the sprinter ran in one minute
b. None of the above
c. The sprinter's average velocity in one minute
d. The sprinter's distance from the starting point after one minute

____ 135. 5.3.2 [P] Suppose f is a diferentiable function. Then $\int_0^x f'(t)dt = f(x)$. Justify your answer.

a. Never
b. Sometimes
c. Always

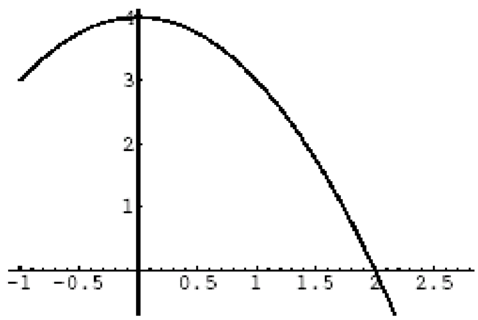
____ 136. 5.3.5 [P] Suppose the function $f(t)$ is continuous and always positive. If G is an antiderivative of f, then we know that G:

a. is always positive.
b. is sometimes positive and sometimes negative.
c. is always increasing.
d. There is not enough information to conclude any of the above.

____ 137. 5.4.1 [Q] If f is continuous and $f(x) < 0$ for all $x \in [a, b]$, then $\int_a^b f(x)dx$

a. must be negative
b. might be 0
c. not enough information

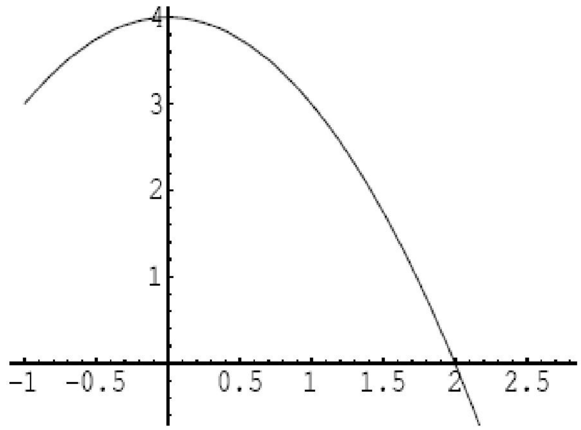
____ 138. 5.4.4 [P] Below is the graph of a function f.



Let $g(x) = \int_0^x f(t)dt$. Then for $0 < x < 2$, $g(x)$is

a. decreasing and concave up.
b. increasing and concave up.
c. decreasing and concave down.
d. increasing and concave down.

____ 139. 5.4.5 [P] Below is the graph of a function f.



Let $g(x) = \int_0^x f(t)dt$. Then

a. $g(0) = 0$, $g'(0) = 0$ and $g'(2) = 0$
b. $g(0) = 1$, $g'(0) = 0$ and $g'(2) = 1$
c. $g(0) = 0$, $g'(0) = 4$ and $g'(2) = 0$
d. $g(0) = 0$, $g'(0) = 0$ and $g'(2) = 1$

____ 140. 5.4.6 [P] You are traveling with velocity $v(t)$ that varies continuously over the interval $[a, b]$ and your position at time t is given by $s(t)$. Which of the following represent your average velocity for that time interval:

(I) $\dfrac{\int_a^b v(t)dt}{b-a}$

(II) $\dfrac{s(b)-s(a)}{b-a}$

(III) $v(c)$ for at least one c between a and b

a. I, II, and III
b. I only
c. I and II only

____ 141. 5.5.1 [Q] The differentiation rule that helps us understand why the Substitution rule works is:

a. The chain rule.
b. The product rule
c. Both of the above.

____ 142. 5.5.2 [D] The area of a circular cell changes as a function of its radius, r, and its radius changes with time $r = g(t)$. If $\dfrac{dA}{dr} = f(r)$, then the total change in area, ΔA between $t = 0$ and $t = 1$ is

a. $\Delta A = \int_{g(0)}^{g(1)} f(r)\,dr$

b. $\Delta A = \int_0^1 f\big(g(t)\big)g'(t)\,dt$

c. $\Delta A = \int_{\pi(g(0))^2}^{\pi(g(1))^2} dA$

d. all of the above

____ 143. 5.5.3 [P] The radius, r, of a circular cell changes with time t. If $r(t) = \ln(t+2)$r(t) = ln(t + 2), which of the following represent the change in area, ΔA of the cell that occurs between $t = 0$ and $t = 1$?

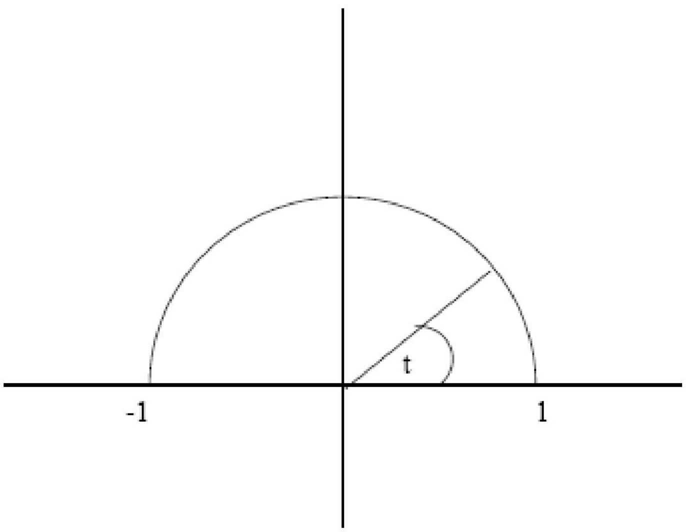
a. $\Delta A = \int_0^1 2\pi \dfrac{\ln(t+2)}{t+2}\,dt$

b. $\Delta A = \pi(\ln 3)^2 - \pi(\ln 2)^2$

c. all of the above

d. $\Delta A = \int_{\ln 2}^{\ln 3} 2\pi r\,dr$

____ 144. 5.5.4 [P] One way to compute $\dfrac{1}{2}$ area of the unit circle is to integrate $\int_{-1}^{1} \sqrt{1-x^2}\,dx$.



Let t be the angle shown. Then the area of the half circle is

a. $\int_{\pi}^{0} -\cos t\,dt$

b. $\int_0^{\pi} -\sin t\,dt$

c. $\int_{\pi}^{0} -\sin^2 t\,dt$

d. $\int_0^{\pi} -\sin^2 t\,dt$

Problem

145. 5.1.4 [Activity] A boneless baked turkey breast that is ten inches long from one end to the other is sliced up in to very thin slices. Each slice has a cross-sectional area of $(-x^2 + 10x)$ square inches for each x between *0* and *10*. What is the volume of the turkey breast?

146. 5.1.5 [Activity] Suppose you slice the carrot the long way? What shape slices would you expect (approximately)? How could you set up an expression for the volume of the whole carrot?

Good Questions for Calculus Original questions by Maria Terrell, http://www.math.cornell.edu/~GoodQuestions/
Answer Section

TRUE/FALSE

1. ANS: F
   Note that even if the two functions have the same rule, they are defined on different domains, i.e., f is not defined at 2.

   PTS: 1 DIF: Calculus 1 REF: 2.1
   OBJ: The tangent and velocity problems and precalculus KEY: domain, equality of functions
   MSC: 398
2. ANS: F
   As x increases to 100, $f(x) = \dfrac{1}{x}$ gets closer and closer to $\dfrac{1}{1000}$ but not as close as to $\dfrac{1}{100}$. The question points out the weakness of the statement "$f(x)$ gets closer to L as $x \to a$, and therefore $\lim\limits_{x \to a} f(x) = L$.

   PTS: 1 DIF: Calculus 1 REF: 2.1 OBJ: The limit of a function
   KEY: limit, intuition MSC: 000
3. ANS: F
   Going to the limit is not monotonic! As a counterexample you can consider
   $$f(x) = \left\{ \begin{array}{ll} 2x & x \ge 0 \\ -x & x < 0 \end{array} \right\}$$
   Then $\lim\limits_{x \to 0} f(x) = 0$, and take $x_1 = 0.25$, $x_2 = -0.35$.

   PTS: 1 DIF: Calculus 1 REF: 2.2 OBJ: The limit of a function
   KEY: limit, monotonic MSC: 552
4. ANS: F
   The goal is to see whether the students understand that it's not enough to check the limit for one particular sequence of numbers that goes to 0. The instructor may want to recall the function $\sin(x)$ from Stewart, as x goes to 0, in order to discuss the problem. Make sure to point out this problem as an example of the danger of using calculators to "find" limits.

   PTS: 1 DIF: Calculus 1 REF: 2.2 OBJ: The limit of a function
   KEY: limit, intuition MSC: 469

5. ANS: F

Most students will answer correctly. However, explain how no matter how close t_0 is to T, $P(t_0)$ might not be close to $P(t)$.

PTS: 1 DIF: Calculus 1 REF: 2.4 OBJ: Continuity
KEY: continuity, discrete functions MSC: 248

6. ANS: T

All students were less than 3 ft tall when they were born, and they should be taller than 3 ft, so by IVT, assuming their growth was continuous, at some point in their life they must have been 3 ft tall. Stress that students may not know WHEN exactly they were 3 feet tall. Instructors should allow the possibility that growth is not continuous, for example some students may reason that they grew by a molecule at a time.

PTS: 1 DIF: Calculus 1 REF: 2.4 OBJ: Continuity
KEY: continuity, IVT, Intermediate Value Theorem MSC: 542

7. ANS: T

Students must consider the difference of two functions: $f(t) = \text{height}(t) - \text{weight}(t)$, functions of time. At birth, $f(\text{birth}) > 0$ and right now, $f(\text{now}) < 0$, hence by IVT $f(\text{T}) = 0$, where T is some time in the past. It is important to stress that this technique of looking at the difference of two functions is recurrent in calculus.

PTS: 1 DIF: Calculus 1 REF: 2.4 OBJ: Continuity
KEY: continuity, IVT, Intermediate Value Theorem MSC: 398

8. ANS: T

Do not use this problem unless you have already worked through the previous two problems. This problem is not at all intuitive. Again, students must consider a difference: the difference in temperature between the diametrically opposite sites.

PTS: 1 DIF: Calculus 1 REF: 2.4 OBJ: Continuity
KEY: continuity, IVT, Intermediate Value Theorem MSC: 415

9. ANS: F

Scoring in basketball is not continuous, so the IVT does not apply here. Students will probably enjoy thinking about this problem.

PTS: 1 DIF: Calculus 1 REF: 2.4 OBJ: Continuity
KEY: continuity, discrete functions, IVT, Intermediate Value Theorem
MSC: 482

10. ANS: T

This problem is not a direct application of IVT, plugging in 0 and 2, we get positive numbers, so the student must choose some other number in $[0,2]$ to test. Choosing 0 and 1, or 1 and 2, IVT immediately applies.

PTS: 1 DIF: Calculus 1 REF: 2.4 OBJ: Continuity
KEY: continuity, IVT, Intermediate Value Theorem MSC: 000

11. ANS: T
    This problem should not be that hard if students have seen this function before.

    PTS: 1 DIF: Calculus 1 REF: 2.4 OBJ: Continuity
    KEY: continuity, limit MSC: 552
12. ANS: F
    Students might justify a True answer by ”zero times any number equals zero”. Point out that it is possible that $\lim_{x \to a} g(x) = \infty$. A quick counterexample can be $a = 0$, $f(x) = x$ and $g(x) = \dfrac{1}{x}$.

    PTS: 1 DIF: Calculus 1 REF: 2.5 OBJ: Limits involving infinity
    KEY: limit, infinity, zero times zero MSC: 469
13. ANS: F
    Students might be thinking that ∞ is a number, and therefore $\infty - \infty = 0$. As a quick counterexample, consider $f(x) = x^2$ and $g(x) = x$.

    PTS: 1 DIF: Calculus 1 REF: 2.5 OBJ: Limits involving infinity
    KEY: limits, infinity, infinity minus infinity MSC: 432
14. ANS: T
    It is easy to sketch a function that crosses its horizontal asymptote.
    For example, consider $\dfrac{\sin x}{x}$.

    PTS: 1 DIF: Calculus 1 REF: 2.5 OBJ: Limits involving infinity
    KEY: limit, infinity, asymptote, horizontal asymptote MSC: 248
15. ANS: T
    This is an easy check.

    PTS: 1 DIF: Calculus 1 REF: 2.7 OBJ: Derivatives
    KEY: derivative, differentiability and continuity MSC: 535
16. ANS: F
    $f'(0)$ equals the slope of the tangent line at $(0,0)$, which is vertical.

    PTS: 1 DIF: Calculus 1 REF: 2.7 OBJ: Derivatives
    KEY: derivative, tangent line, slope MSC: 398
17. ANS: T
    This gets students to think about tangent lines and derivatives. A vertical tangent line exists, although the derivative does not.

    PTS: 1 DIF: Calculus 1 REF: 2.7 OBJ: Derivatives
    KEY: derivative, tangent line, slope MSC: 541

18. ANS: F

The limit $\lim_{x \to 0} \frac{f(x) - 0}{x - 0}$ does not exist, because it equals -1 from the left, and 1 from the right. Thus $f'(0)$ does not exist.

PTS: 1 DIF: Calculus 1 REF: 2.7 OBJ: Derivatives
KEY: derivatives, continuity, absolute value, tangent line MSC: 448

19. ANS: T

Easy application of the limit definition of derivative. Students should note that close to 1, $|x| = x$. $g'(1) = 2$

PTS: 1 DIF: Calculus 1 REF: 2.7 OBJ: Derivatives
KEY: derivatives, continuity, absolute value, tangent line MSC: 248

20. ANS: F

Point out that for n-even and all n-odd, $n > 1$, the equation holds; but for all other n, $|x|^n$ is not differentiable at zero.

PTS: 1 DIF: Calculus 1 REF: 3.1
OBJ: Derivatives of polynomials and exponential functions
KEY: derivatives, continuity, absolute value MSC: 432

21. ANS: F

Some students will be unable to recognize that we get the same result. Remind them that the derivative of a constant function is zero.

PTS: 1 DIF: Calculus 1 REF: 3.2 OBJ: The product and quotient rules
KEY: product rule, derivative of a constant, constant rule MSC: 398

22. ANS: F

This is a nice preview of Chain Rule, also good exercise for a deep understanding of the limit definition of derivative. To be compared with the previous problem.

PTS: 1 DIF: Calculus 1 REF: 3.4
OBJ: Derivatives of trigonometric functions
KEY: chain rule, trigonometric functions, definition of derivative
MSC: 398

23. ANS: F

Students must observe that $\ln(\pi)$ is a constant, and thus $\frac{d}{dx} \ln(\pi) = 0$.

PTS: 1 DIF: Calculus 1 REF: 3.7 OBJ: Derivatives of logarithmic functions
KEY: product rule, derivative of a constant, constant rule MSC: 448

24. ANS: F

$\frac{f'}{f}$ is the derivative of $\ln|f|$. Students should check by differentiating.

PTS: 1 DIF: Calculus 1 REF: 3.7 OBJ: Derivatives of logarithmic functions
KEY: derivative, logarithmic functions, chain rule MSC: 248

25. ANS: T

The answer to this problem becomes apparent after constructing the mathematical model. Using the Pythagorean theorem, one can get $x\frac{\mathrm{d}x}{\mathrm{d}t}=z\frac{\mathrm{d}z}{\mathrm{d}t}$ where z is the length of the rope, and x is the distance to the dock, as in the previous problem. So writing $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{z}{x}\frac{\mathrm{d}z}{\mathrm{d}t}$ we see that as $x\to 0$, $\frac{\mathrm{d}x}{\mathrm{d}t}$ increases. In fact, according to our model the boat will, at some point, be traveling faster than the speed of light! It will be interesting for the students to discuss why our model fails to describe what actually happens.

PTS: 1 DIF: Calculus 1 REF: 4.1 OBJ: Related Rates
KEY: related rates, pythagorean theorem MSC: 000

26. ANS: F

Encourage the students to think of the different ways in which this would fail, i.e. at the endpoints or points where the derivative does not exist.

PTS: 1 DIF: Calculus 1 REF: 4.2 OBJ: Maximum and Minimum Values
KEY: derivative, maxima, minima, endpoints, cusps, extreme values
MSC: 432

27. ANS: F

Students tend to believe this is a true statement. Point out an example when the statement does not hold, $f(x)=x^4$ at $a=0$ for example.

PTS: 1 DIF: Calculus 1 REF: 4.3
OBJ: Mean Value Theorem and shapes of curves
KEY: derivative, maxima, minima, endpoints, cusps, extreme values
MSC: 560

28. ANS: F

This emphasizes the differentiability hypotheses when using the MVT (Note that $f(x)$ is not differentiable at 0).

PTS: 1 DIF: Calculus 1 REF: 4.3
OBJ: Mean Value Theorem and shapes of curves
KEY: derivative, MVT, Mean Value Theorem MSC: 398

29. ANS: T

It might be a bit hard for students to think of the model, but this problem is a good preparation for the much harder problem that follows. Note that the IVT could have also been used, by arguing that the velocity was during one part of the race positive and then it became negative.

PTS: 1 DIF: Calculus 1 REF: 4.3
OBJ: Mean Value Theorem and shapes of curves
KEY: derivative, MVT, Mean Value Theorem, IVT MSC: 248

30. ANS: T

Consider a function with a cusp at $x = a$ and $f(a) = 1$, which remains positive, and decreases to a horizontal asymptote at $y = 0$.

PTS: 1 DIF: Calculus 1 REF: 4.6 OBJ: Optimization
KEY: derivative, maxima, minima, endpoints, cusps, extreme values
MSC: 541

31. ANS: F

The goal is to test whether students understand that they need one initial condition for each antiderivative they have to find.

PTS: 1 DIF: Calculus 1 REF: 4.9 OBJ: Antiderivatives
KEY: antiderivative, velocity, acceleration, initial condition MSC: 343

32. ANS: F

$f(x)$ is not continuous at 0! This problem may help them to remember the continuity condition in finding antiderivatives. Students are almost surely going to answer true, and the answer will surprise them.

PTS: 1 DIF: Calculus 1 REF: 4.9 OBJ: Antiderivatives
KEY: antiderivative, continuity, initial condition MSC: 248

33. ANS: T

This is an easy check based on the text reading.

PTS: 1 DIF: Calculus 1 REF: 4.9 OBJ: Antiderivatives
KEY: antiderivative, sums of functions MSC: 494

34. ANS: F

Students should differentiate an antiderivative of f times an antiderivative of g, to see that they will not get fg back.

PTS: 1 DIF: Calculus 1 REF: 4.9 OBJ: Antiderivatives
KEY: antiderivatives, products of functions MSC: 541

35. ANS: T

This problem helps students to use the summation notation to represent physical quantities. It also gets the students to distinguish between the estimating procedure of a Riemann Sum versus cases in which we can have an exact value.

PTS: 1 DIF: Calculus 1 REF: 5.1 OBJ: Areas and Distances
KEY: summation notation, delta notation MSC: 398

36. ANS: F

This problem will remind the students that when we are taking the limit as n goes to infinity, we can choose any sample points that we want. Instructors should point out that even though the limits are the same, the corresponding sequences for approximations are not equally good.

PTS: 1 DIF: Calculus 1 REF: 5.2 OBJ: The Definite Integral
KEY: summation notation, delta notation, limit, sequences MSC: 000

37. ANS: T

This problem emphasizes the difference between definite and indefinite integrals.

PTS: 1 DIF: Calculus 1 REF: 5.2 OBJ: The Definite Integral
KEY: integral, definite integral MSC: 552

38. ANS: F

See below.

PTS: 1 DIF: Calculus 1 REF: 5.3 OBJ: Evaluating Definite Integrals
KEY: integral, indefinite integral, derivative, antiderivative, additive constant
MSC: 398

39. ANS: F

As students often get confused in the mechanics of the process of going back and forth between functions, their derivatives and antiderivatives, a discussion using the above problem can help them clarify their misunderstandings.

PTS: 1 DIF: Calculus 1 REF: 5.3 OBJ: Evaluating Definite Integrals
KEY: integral, indefinite integral, derivative, antiderivative, additive constant
MSC: 343

40. ANS: T

This is an immediate application of the Extreme Value Theorem. It is easily seen via an area argument (fitting the graph of f inside a box).

PTS: 1 DIF: Calculus 1 REF: 5.4
OBJ: The Fundamental Theorem of Calculus
KEY: FTC, Fundamental Theorem of Calculus, EVT, Extreme Value Theorem, bounding boxes
MSC: 248

41. ANS: F

Students often do not realize that definite integrals evaluated at constant endpoints a and b are constant, and in order to apply the FTC one must have one at least one of the endpoints as a variable. Note that if a and/or b were defined as functions of x, then the answer would be True.

PTS: 1 DIF: Calculus 1 REF: 5.4
OBJ: The Fundamental Theorem of Calculus
KEY: FTC, Fundamental Theorem of Calculus, antiderivative, definite integral
MSC: 541

MULTIPLE CHOICE

42. ANS: C

Both f and g are given by the same rule, and are defined on the same domain, hence they are the same function.

PTS: 1 DIF: Calculus 1 REF: 2.1
OBJ: The tangent and velocity problems and precalculus KEY: precalculus

43. ANS: D

This question is quite difficult for students because it is very counterintuitive. A little algebra needs to be done to see that as long as the student is not over $\frac{20}{2\pi}$ meters tall, she should be able to walk under the rope. Students should know or be provided with the perimeter of a circle. There is no need to know the radius of the Earth at equator. The problem encourages using a mathematical model to check one's intuition. Instructors should validate students' intuition: the change in radius is very small relative to the radius, and this may lead to the erroneous conclusion that a human would not be able to walk underneath the rope; however, a human's height is also very small relative to the radius.

PTS: 1 DIF: Calculus 1 REF: 2.1
OBJ: The tangent and velocity problems and precalculus MSC: 415

44. ANS: A

Students may be unsure about real numbers as infinite decimals. Students know that all rational numbers have terminating or repeating decimal representations. They also know that there are irrational numbers, hence there are some numbers that are represented as infinite decimals. However, they may not know that every infinite decimal represents a number (although not uniquely in the case of repeating 9s and repeating 0s) -The phrase can be defined precisely may cause some to reject this as a solution. In discussing this question, instructors can introduce the idea that every infinite decimal is a number and the Archimedian Axiom can help us see how we can tell whether two numbers are the same.

PTS: 1 DIF: Calculus 1 REF: 2.1
OBJ: The tangent and velocity problems and precalculus MSC: 482

45. ANS: B

If using this problem, the instructor should briefly talk about the Archimedian Axiom, and how intersection of nested closed intervals I_n of respective lengths $\frac{1}{n}$, is a single point. Since both a and b are in each of these I_n, this single point of intersection is $a = b$. Students have a hard time understanding the Squeeze Theorem, so this might be a good place to start in attacking that problem.

PTS: 1 DIF: Calculus 1 REF: 2.2 OBJ: The limit of a function
KEY: limits, Squeeze Theorem MSC: 432

46. ANS: D
Students should be encouraged to draw the graph and discuss.

PTS: 1 DIF: Calculus 1 REF: 2.2 OBJ: The limit of a function
MSC: 535

47. ANS: C
Use this problem to stress that $f(a)$ need not be defined in order for $\lim_{x \to a} f(x)$ to exist. Students have a difficult time asserting "never". The problem provides an opportunity to discuss what a limit is.

PTS: 1 DIF: Calculus 1 REF: 2.3
OBJ: Calculating limits using the limit laws KEY: limit, definition of limit
MSC: 448

48. ANS: A
Answers "cannot" and "must" are very popular. $f(a)$ need not be defined in order for $\lim_{x \to a} f(x)$ to exist, and it does not have to approach 1. However, the limit could be 0, for example consider $f(x) = 0$ for all $x \neq a$, and $f(x)$ not defined. The student has to note the difference between "cannot", "could" and "must".

PTS: 1 DIF: Calculus 1 REF: 2.3
OBJ: Calculating limits using the limit laws KEY: limit, definition of limit, domain
MSC: 000

49. ANS: B
Point out that $\frac{0}{0}$ is not always equal to 1. If this question is used after any of the previous two problems, more students will be able to answer correctly.

PTS: 1 DIF: Calculus 1 REF: 2.3
OBJ: Calculating limits using the limit laws KEY: limit, zero divided by zero
MSC: 612

50. ANS: B
Illustrate why (b) and (c) are not the reason why the limit does not exist, by introducing the next problem.

PTS: 1 DIF: Calculus 1 REF: 2.3
OBJ: Calculating limits using the limit laws MSC: 612

51. ANS: E
As in the previous problem, the function oscillates and 1/0 is undefined, however, this limit exists.
This is also a nice application of The Squeeze Theorem:
$$\lim_{x \to 0}\left(-x^2\right) \leq \lim_{x \to 0} x^2 \sin(1/x) \leq \lim_{x \to 0} x^2$$
Therefore, the limit equals 0.

PTS: 1 DIF: Calculus 1 REF: 2.3
OBJ: Calculating limits using the limit laws MSC: 409

52. ANS: C

This problem requires a geometrical argument:

Solution 1: By similar triangles, $\dfrac{f(x)}{6} = \dfrac{x-a}{0-a} = \dfrac{g(x)}{3}$, and therefore $\dfrac{f(x)}{g(x)} = \dfrac{6}{3} = 2$

Solution 2: $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to a} \dfrac{\dfrac{f(x)}{-a}}{\dfrac{g(x)}{-a}} = \lim\limits_{x \to a} \dfrac{\text{slope of } f}{\text{slope of } g} = \dfrac{6}{3} = 2$

This problem is a nice preview of L'Hospital's Rule.

PTS: 1 DIF: Calculus 1 REF: 2.3
OBJ: Calculating limits using the limit laws MSC: 343

53. ANS: A

Students should be encouraged to draw $f(t)$ and should be able to see the answer quickly. Note that (a) can also be the correct answer, depending on the model that students use for the phenomenon: if the drop of water gradually merges with the water in the tub, the function is continuous with respect to time.

PTS: 1 DIF: Calculus 1 REF: 2.4 OBJ: Continuity
KEY: continuity MSC: 560

54. ANS: A

Again, students should be encouraged to draw the graph of $g(x)$. It is interesting to compare this to the previous question. It should be pointed out the difference between the independent variables in the two problems.

PTS: 1 DIF: Calculus 1 REF: 2.4 OBJ: Continuity
MSC: 494

55. ANS: C

This may seem like an easy logic question, but students tend to have difficulties; it might be a good time to review some logic. This question prepares students for reasoning that even if differentiability implies continuity, continuity does not imply differentiability. Ask for examples of functions that are continuous, but not polynomials.

PTS: 1 DIF: Calculus 1 REF: 2.4 OBJ: Continuity
MSC: 541

56. ANS: B

First recall problem 4 in section 2.1, or present that problem if you have not used it. Point out that we estimate e^2 by considering the limit $\lim\limits_{x \to e} x^2$ as opposed to $\lim\limits_{x \to 2} e^x$.

PTS: 1 DIF: Calculus 1 REF: 2.4 OBJ: Continuity
MSC: 533

57. ANS: A

Recall problem 6. in Section 2.3. $\dfrac{f(x)}{6}=\dfrac{x-a}{0-a}=\dfrac{g(x)}{3}$ and therefore $\dfrac{f(x)}{g(x)}$ is $\dfrac{6}{3}=2$.

PTS: 1 DIF: Calculus 1 REF: 2.5 OBJ: Limits involving infinity
MSC: 541

58. ANS: B

Students must pay attention to the way horizontal asymptotes are defined. Point out that asymptotes are defined as we go to ∞ and to $-\infty$, even though a function may have asymptotic behavior at other points.

PTS: 1 DIF: Calculus 1 REF: 2.5 OBJ: Limits involving infinity
MSC: 533

59. ANS: D

The limit $\lim\limits_{x\to 0}\dfrac{f(x)-0}{x-0}$ does not exist, because it equals -1 from the left, and 1 from the right. Thus, we have NO tangent line at $(0,0)$. Students might choose any of the three answers, based on their high-school experience with tangent lines. Stress that a tangent line is not necessarily a line that touches the curve at only one point (as in the case of the circle); also a curve has a tangent line at some point $(a,f(a))$ if "zooming in " on the curve at this point, the curve looks like a line. This problem is an example of a function with infinitely many lines touching at one point, but no tangent line.

PTS: 1 DIF: Calculus 1 REF: 2.6
OBJ: Tangents, velocities, and other rates of change MSC: 482

60. ANS: B

Before doing the math, students might be inclined to choose any one of the answers, based on their high-school experience with tangents. Using the definition of the tangent line, they must see that the tangent at $(0,0)$ is a line of slope 1, passing thru $(0,0)$. This is a problem that illustrates the existence of functions with tangent lines touching the function at infinitely many points.

PTS: 1 DIF: Calculus 1 REF: 2.6
OBJ: Tangents, velocities, and other rates of change KEY: tangents, tangent lines
MSC: 612

61. ANS: B

This is a straightforward application of the definition of average rate of change. Once we get (c) as an answer, (a) and (b) follow:

$$\frac{\pi r\frac{2}{2}-\pi r\frac{2}{1}}{r_2-r_1}=\pi\left(r_2+r_1\right)=2\pi\frac{r_2+r_1}{2}$$

PTS: 1 DIF: Calculus 1 REF: 2.6
OBJ: Tangents, velocities, and other rates of change KEY: rates of change
MSC: 409

62. ANS: B

Students should be encouraged to think about tangent lines and the definition of derivative to think about this question.

PTS: 1 DIF: Calculus 1 REF: 2.6
OBJ: Tangents, velocities, and other rates of change
KEY: tangents, derivatives, derivative at a point MSC: 343

63. ANS: C

Since $V = \pi r^2 h$ and r is constant, V and h are proportional to each other, so that $\dfrac{\Delta V}{\Delta h} = \dfrac{\pi r^2 \Delta h}{\Delta h} = \pi r^2$, is constant.

PTS: 1 DIF: Calculus 1 REF: 2.6
OBJ: Tangents, velocities, and other rates of change KEY: rates of change
MSC: 248

64. ANS: A

Instructors should encourage the use of the limit definition of derivative; $f'(0) = 0$

PTS: 1 DIF: Calculus 1 REF: 2.7 OBJ: Derivatives
MSC: 398

65. ANS: B

If f is differentiable at a, it must be continuous at a, and therefore the limit equals $f(a)$. Many students will like answers (a) and (b).

PTS: 1 DIF: Calculus 1 REF: 2.8 OBJ: The derivative as a function
KEY: derivative function, differentiability and continuity MSC: 415

66. ANS: B

This is a nice logic question. Ask students to give examples of continuous functions that are not differentiable. If you have done some other logic questions, this will probably be an easy question, otherwise, a short introduction/review to logic might be necessary.

PTS: 1 DIF: Calculus 1 REF: 2.8 OBJ: The derivative as a function
KEY: differentiability and continuity, logic MSC: 482

67. ANS: D

Once the students realize that $f'(x)$ is the speed of the train after x hours, most of them will give the right answer.

PTS: 1 DIF: Calculus 1 REF: 2.8 OBJ: The derivative as a function
MSC: 000

68. ANS: E

Students will know that I holds. They might have questions about I and not be quite sure about I.

PTS: 1 DIF: Calculus 1 REF: 2.8 OBJ: The derivative as a function
KEY: differentiability and continuity MSC: 552

69. ANS: A

Quick check. Students must see that e^7 is a constant.

PTS: 1 DIF: Calculus 1 REF: 3.1
OBJ: Derivatives of polynomials and exponential functions MSC: 469

70. ANS: B

Recall the limit definition of the derivative; then one can easily check (b). Some may be inclined to answer (a); also, some students might be able to get the right answer, 10, by factoring $x^{10} - 1$, and then simplifying. Answer (c) comes from a cancellation error that students may make.

PTS: 1 DIF: Calculus 1 REF: 3.1
OBJ: Derivatives of polynomials and exponential functions MSC: 541

71. ANS: A

The area of the slice of pizza varies as the angle varies. The slice of pizza has area $A(\theta) = \frac{1}{2} r^2 \theta$.

Hence $A'(\theta) = \frac{1}{2} r^2$. Students should pay attention to the variable with respect to which they differentiate. Instructors may want to encourage their students to use the limit definition of the derivative in this problem.

PTS: 1 DIF: Calculus 1 REF: 3.1
OBJ: Derivatives of polynomials and exponential functions MSC: 448

72. ANS: D

Quick check. This type of problem prepares students for later related rates problems.

PTS: 1 DIF: Calculus 1 REF: 3.1
OBJ: Derivatives of polynomials and exponential functions MSC: 542

73. ANS: C

This may be a difficult question for students: it will be hard to figure out which quantity we need to differentiate, and with respect to what. Instructors should point out the difference between $(a) - (c)$.

PTS: 1 DIF: Calculus 1 REF: 3.1
OBJ: Derivatives of polynomials and exponential functions MSC: 494

74. ANS: B

Straight forward application of quotient rule: $\left.\frac{d}{dx}\frac{f(x)}{x^2}\right|_{x=1} = \frac{f'(1) - 2f(1)}{1} = 1$.

PTS: 1 DIF: Calculus 1 REF: 3.2 OBJ: The product and quotient rules
KEY: 612 MSC: 612

75. ANS: B
The answer follows from an easy computation:

$$\frac{\Delta A}{\Delta t} = \frac{L(\Delta W) + W(\Delta L) + (\Delta L)(\Delta W)}{\Delta t} = L\frac{\Delta W}{\Delta t} + W\frac{\Delta L}{\Delta t} + \frac{(\Delta W)(\Delta L)}{\Delta t}$$

PTS: 1 DIF: Calculus 1 REF: 3.2 OBJ: The product and quotient rules
MSC: 409

76. ANS: D
Students may be more comfortable using (b)or (c) as answers, even though they are wrong. This kind of problem should help them make the connection between $\frac{dL}{dt}$ and $\frac{\Delta L}{\Delta t}$

$$\frac{\Delta A}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta A}{\Delta t} = \lim_{\Delta t \to 0}\left[L\frac{\Delta W}{\Delta t} + W\frac{\Delta L}{\Delta t} + \Delta W\frac{\Delta L}{\Delta t}\right] = L\frac{dW}{dt} + W\frac{dL}{dt} + 0\cdot\frac{dL}{dt}$$

PTS: 1 DIF: Calculus 1 REF: 3.2 OBJ: The product and quotient rules
MSC: 343

77. ANS: D
Note that this is the derivative of $\sin x$ at *0*, that is, the slope of the function $\sin x$ at 0 is 1. So the tangent to the graph $(0,0)$ is the line $y = x$. Students will have trouble identifying the limit as a derivative, and also thinking of the derivative at 0 as the slope of the function (and of the tangent) at 0.

PTS: 1 DIF: Calculus 1 REF: 3.4
OBJ: Derivatives of trigonometric functions MSC: 560

78. ANS: D
This problem offers a chance to highlight some of the interesting periodic properties of the derivative of $\sin x$.

PTS: 1 DIF: Calculus 1 REF: 3.4
OBJ: Derivatives of trigonometric functions MSC: 494

79. ANS: E
(a) - (c) are equivalent formulae. One can get (a) by using the product rule and $\cos^2 x = 1 - \sin^2 x$.

PTS: 1 DIF: Calculus 1 REF: 3.4
OBJ: Derivatives of trigonometric functions MSC: 541

80. ANS: C
$f'(x) = \sec^2(x) = 1 + \tan^2(x)$. The other answers may arise if students compute the derivative in the following way: $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x}{-\sin x}$.

PTS: 1 DIF: Calculus 1 REF: 3.4
OBJ: Derivatives of trigonometric functions MSC: 533

81. ANS: D

$\lim\limits_{h \to 0} \dfrac{\sin(2x+h)-\sin(2x)}{h} = \dfrac{d}{dy}(\sin y)|_{y=2x}$. This problem is making the connection between differentiation rules and the limit definition of derivative.

PTS: 1 DIF: Calculus 1 REF: 3.4
OBJ: Derivatives of trigonometric functions MSC: 542

82. ANS: A

There is not enough information since the missing factor, $\dfrac{dx}{dr}$ could be anything. If $x = r$, then $\cos x + e^{\sin x}\cos x$ is correct. If x is constant with respect to r, then the answer is 0. Students may have difficulty considering $(\sin x + e^{\sin x})$ as a constant with respect to r.

PTS: 1 DIF: Calculus 1 REF: 3.5 OBJ: Chain Rule
KEY: chain rule MSC: 415

83. ANS: C

Straight forward application of chain rule if they have seen this type of problem before.

PTS: 1 DIF: Calculus 1 REF: 3.5 OBJ: Chain Rule
MSC: 482

84. ANS: C

Even though students may have memorized the Chain Rule formula, some may not be able to apply it to this type of problem.

PTS: 1 DIF: Calculus 1 REF: 3.5 OBJ: Chain Rule
MSC: 000

85. ANS: B

This is an easy application of the Chain Rule; it prepares students for Related Rates problems.

PTS: 1 DIF: Calculus 1 REF: 3.5 OBJ: Chain Rule
MSC: 552

86. ANS: B

All equations in x and y can be written as a sum of terms of the form $f(x)g(y)$, and hence using implicit differentiation on this term,

$$\frac{d}{dx} f(x)g\left(y\right) = f'(x)g(y) + f(x)\frac{d}{dy}g(y)\cdot y'$$

hence no term in the sum has factors of the form $\left(y'\right)^2$. Therefore, we never have to solve a quadratic. Students might be most comfortable answering (b), and if so, ask them first if they ever had to solve a quadratic while using implicit differentiation, and then go into more detail to why this does not happen.

PTS: 1 DIF: Calculus 1 REF: 3.6 OBJ: Implicit Differentiation
KEY: Implicit Differentiation MSC: 469

87. ANS: B

This example gives the instructor the opportunity to stress that $(\sin x)^{-1}$ is not the same as $\sin^{-1} x$.

PTS: 1 DIF: Calculus 1 REF: 3.6 OBJ: Implicit Differentiation
MSC: 432

88. ANS: A

Using implicit differentiation, $1 = \cos y \dfrac{dy}{dx}$. So at $(0, \pi)$, the slope is $\sec \pi$ = -1. Students should note that even though this is not the graph of a function, it still has a tangent line at this point.

PTS: 1 DIF: Calculus 1 REF: 3.6 OBJ: Implicit Differentiation
MSC: 541

89. ANS: B

e can be approximated very well by taking a number that is extremely close to 1, and raising it to a high enough power. Students may be puzzled by this answer, since $\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \ldots., \frac{101}{100}, \ldots$ gets very close to 1, and "1 to any power is 1". This may be a good time to point out that 1^{∞} is not defined. Note that (*a*) and (*b*) are also correct answers with the appropriate meanings, i.e., *e* is not a number in the sense that it cannot be written down or completely computed numerically.

PTS: 1 DIF: Calculus 1 REF: 3.7 OBJ: Derivatives of logarithmic functions
MSC: 448

90. ANS: C

Most students will be able to get a correct formula for the linearization of e^x at 0, $L(x) = 1 + x$, but a good number will have difficulties knowing how to use this information to get an approximation for $e^{.5}$.

PTS: 1 DIF: Calculus 1 REF: 2.9 & 3.8
OBJ: Linear approximations and Differentials KEY: linear approximations, differentials
MSC: 612

91. ANS: C

If presented in the Linearization Approximation section, this problem will be straightforward for most students.

PTS: 1 DIF: Calculus 1 REF: 2.9 & 3.8
OBJ: Linear approximations and Differentials MSC: 409

92. ANS: A

Students might be inclined to answer (*a*), since usually we use tangent line approximations for values of *x* close to *a*; or they might choose (*c*), since the closer we are to *a*, the better the approximation will be, without realizing that we might get a good approximation even far from *a*.

PTS: 1 DIF: Calculus 1 REF: 2.9 & 3.8
OBJ: Linear approximations and Differentials MSC: 343

93. ANS: A

Students might need a hint: plug in the formula for $\mathrm{E}(x)$ and then split the limit into two limits that you recognize and can compute.

$$\lim_{x \to a} \frac{\mathrm{E}(x)}{(x-a)} = \lim_{x \to a} \frac{f(x)-f(a)}{(x-a)} - \lim_{x \to a} \frac{f'(a)(x-a)}{(x-a)} = f'(a) - f'(a) = 0$$

PTS: 1 DIF: Calculus 1 REF: 2.9 & 3.8
OBJ: Linear approximations and Differentials MSC: 248

94. ANS: D

Note that $f'(a)$ is equal to the slope of L_1, and $g'(a)$ equals the slope of L_2.

$L_1(x) = f(a) + f'(a)(x-a) = f'(a)(x-a)$ and $L_2(x) = g(a) + g'(a)(x-a) = g'(a)(x-a)$

Using notation from number 4 above,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{L_1(x)+E_1(x)}{L_2(x)+E_2(x)} = \lim_{x \to a} \frac{f'(a)(x-a)+E_1(x)}{g'(a)(x-a)+E_2(x)} = \lim_{x \to a} \frac{f'(a)+\dfrac{E_1(x)}{(x-a)}}{g'(a)+\dfrac{E_2(x)}{(x-a)}} = \frac{f'(a)+0}{g'(a)+0}$$

$$= \frac{f'(a)}{g'(a)} = \frac{slope\_of\_L_1}{slope\_of\_L_2} = 2$$

PTS: 1 DIF: Calculus 1 REF: 2.9 & 3.8
OBJ: Linear approximations and Differentials MSC: 560

95. ANS: A

This will be clear for most students. Ask for a graph of such a function f and its tangent line approximation at some point a.

PTS: 1 DIF: Calculus 1 REF: 2.9 & 3.8
OBJ: Linear approximations and Differentials MSC: 494

96. ANS: B

ΔVrepresents the change in volume, hence it is the volume of the part that is being removed, i.e., the rind. This problem is a nice introduction to differentials. Note that "-1 × (the volume of the rind)" also works and may be a more natural derivation.

PTS: 1 DIF: Calculus 1 REF: 2.9 & 3.8
OBJ: Linear approximations and Differentials MSC: 541

97. ANS: C

If you use this problem following number 2, III clearly does not apply, and IV does not hold because we need an additional constant factor of 4π. Since the skin is very thin, the radius of the orange R is approximately the radius of the edible part r, so are their respective surface areas. More exactly,

$$\frac{\Delta V}{\Delta x} = \frac{4}{3}\pi \frac{R^3 - r^3}{R - r} = \frac{4}{3}\pi\left(R^2 + rR + r^2\right) \approx 4\pi R^2 \left(\approx 4\pi r^2\right)$$

PTS: 1 DIF: Calculus 1 REF: 2.9 & 3.8
OBJ: Linear approximations and Differentials MSC: 533

98. ANS: C

Since $A = x^2$, $dA = 2x\,dx$, where $dx = x_2 - x_1$. Thus, $dA = 2x_1\left(x_2 - x_1\right)$. Some students might think $dA = \triangle A = x_2^2 - x_1^2$ and answer (c). Others may plug in $x = x_2$ and thus answer (b).

PTS: 1 DIF: Calculus 1 REF: 2.9 & 3.8
OBJ: Linear approximations and Differentials MSC: 398

99. ANS: B

Note that $\Delta A = x_2^2 - x_1^2 = \left(x_2 + x_1\right) \cdot \left(x_2 - x_1\right) > 2x_1 \cdot \left(x_2 - x_1\right) = dA$. This can also be shown by taking the second derivative.

PTS: 1 DIF: Calculus 1 REF: 2.9 & 3.8
OBJ: Linear approximations and Differentials MSC: 398

100. ANS: A

Easy application of the Chain Rule + Product Rule.

PTS: 1 DIF: Calculus 1 REF: 4.1 OBJ: Related Rates
KEY: related rates MSC: 415

101. ANS: A

Let $z(t)$ be the length of the rope, and $x(t)$ be the horizontal distance from the boat to the dock. Then, using the Pythagorean Theorem, and differentiating, one gets: $z \cdot \frac{dz}{dt} = x \cdot \frac{dx}{dt}$, and the answer follows.
This problem encourages the students to use their intuition, before writing down the mathematical model, as they start thinking about a related rates problem. A good point can be made about the power of mathematics to supplement or test our intuition. Note that answer “One is a constant multiple of the other.” is correct in the case the boat is pulled in from a floating dock.

PTS: 1 DIF: Calculus 1 REF: 4.1 OBJ: Related Rates
MSC: 482

102. ANS: C

Similar triangles and some knowledge of related rates, make this problem fairly easy to set up. The lack of numbers will help students concentrate on the idea of related rates. It is interesting to contrast this problem and the next one.

PTS: 1 DIF: Calculus 1 REF: 4.1 OBJ: Related Rates
MSC: 552

103. ANS: B

This problem should be done after the one above. It helps students see that very similar problem set ups predict different outcomes.

PTS: 1 DIF: Calculus 1 REF: 4.1 OBJ: Related Rates
MSC: 469

104. ANS: B

Students should think about the differences between local and absolute extrema. Ask students to draw pictures of functions satisfying $f'(a) > 0$, with a being the left endpoint, and then a being the right endpoint.

PTS: 1 DIF: Calculus 1 REF: 4.2 OBJ: Maximum and Minimum Values
KEY: maxima, minima MSC: 541

105. ANS: B

This problem emphasizes one important application of the EVT which is at the same time a very geometric result; that we can put a continuous function on a closed interval inside a box! On a closed interval, $m \le f(x) \le M$, so if we take A = max{$|m|$, $|M|$}, then on this closed interval, f fits in this sort of box. This idea will ultimately show up in finding bounds for integrals, so it would be great to introduce this idea at this point.

PTS: 1 DIF: Calculus 1 REF: 4.2 OBJ: Maximum and Minimum Values
MSC: 448

106. ANS: D

The area of the slice of pizza is the area of a sector of angle θ , $A(\theta) = \frac{1}{2} r^2 \theta$. Hence $A'(\theta) = \frac{1}{2} r^2$. Students should pay attention to the variable with respect to which they differentiate.

PTS: 1 DIF: Calculus 1 REF: 4.2 OBJ: Maximum and Minimum Values
MSC: 533

107. ANS: A

This problem can be used after the instructor talks about absolute extrema in a closed interval. Using the definition of derivative, $A'(\theta) = \lim\limits_{\theta \to 0} \dfrac{\Delta A}{\Delta \theta}$. We know that in a piece of pizza with angle $\Delta\theta$, the radius is always in $[r_{\min}(\theta), r_{\max}(\theta)]$ for some $r_{\min}(\theta)$ and $r_{\max}(\theta)$ (EVT). Hence

$$\frac{\frac{1}{2}(r_{\min}(\theta))^2 \Delta\theta}{\Delta\theta} \le \frac{\Delta A}{\Delta\theta} \le \frac{\frac{1}{2}(r_{\max}(\theta))^2 \Delta\theta}{\Delta\theta}$$

Since as $\Delta\theta \to 0$, $r_{\min}(\theta) < r(\theta)$ and $r_{\max}(\theta) < r(\theta)$, by the Squeeze Theorem, $A'(\theta) = \dfrac{1}{2}(r(\theta))^2$.

PTS: 1 DIF: Calculus 1 REF: 4.2 OBJ: Maximum and Minimum Values
MSC: 542

108. ANS: A

The surface area of the trunk is between $[A_{\min}, A_{\max}]$ during the time in which the tree adds a ring of thickness Δx. Then,

$$\frac{A_{\min}\Delta x}{\Delta x} \le \frac{\Delta V}{\Delta x} \le \frac{A_{\max}\Delta x}{\Delta x}$$

As $\Delta x \to 0$, $A_{\min} \to A$ and $A_{\max} \to A$, where A is the surface area of the trunk.

Therefore, $V'(x) = A(x)$.

PTS: 1 DIF: Calculus 1 REF: 4.2 OBJ: Maximum and Minimum Values
MSC: 612

109. ANS: A

$\dfrac{dV}{dx} = \lim\limits_{\Delta x \to 0} \dfrac{dV}{dx}$. As in the previous two problems, using the EVT, one can bound the surface area A(x) of the last slice by $A_{min} \le A(x) \le A_{max}$. We get

$$\lim_{\Delta x \to 0} \frac{A_{\min}\Delta x}{\Delta x} \le \frac{dV}{dx} \lim_{\Delta x \to 0} \frac{A_{\max}\Delta x}{\Delta x}$$

Since again $A_{\min}$ and $A_{\max}$ approach $A(x)$ as $\Delta x \to 0$, $\dfrac{dV}{dx} = A(x)$.

PTS: 1 DIF: Calculus 1 REF: 4.2 OBJ: Maximum and Minimum Values
MSC: 409

110. ANS: B

If f is the stock price, and we found out that f' is increasing, this does not lead to any conclusion about the monotonicity of f.

PTS: 1 DIF: Calculus 1 REF: 4.3
OBJ: Mean Value Theorem and shapes of curves
KEY: MVT, mean value theorem, shapes of curves MSC: 343

111. ANS: C

Have students relate what they THINK is happening to the mathematical concepts. For example, some students might think that their speed increases at a faster rate, while others will see that the ”terminal velocity” idea points to a slowing in the rate of increase.

PTS: 1 DIF: Calculus 1 REF: 4.3
OBJ: Mean Value Theorem and shapes of curves MSC: 248

112. ANS: A

It is easy to see that the function is increasing: the more water we add, the bigger the height. To see that the function is concave down, observe that the instantaneous rate of change of the height with respect to the volume is decreasing: as the cup gets filled, for a fixed increment in water, we get smaller and smaller increments in height.

PTS: 1 DIF: Calculus 1 REF: 4.3
OBJ: Mean Value Theorem and shapes of curves MSC: 494

113. ANS: D

First answer this question in the easiest form, discussing instantaneous and average velocity - then go back and try to answer again in a way that allows the application of the mean value theorem easily. The fact that (c) is also true, can be shown by using a graph. This is probably the hardest part of the problem.

PTS: 1 DIF: Calculus 1 REF: 4.3
OBJ: Mean Value Theorem and shapes of curves MSC: 541

114. ANS: A

$\dfrac{A_2 - A_1}{r_2 - r_1} = \dfrac{\pi\left(r_2^2 - r_1^2\right)}{r_2 - r_1} = \pi\left(r_2 + r_1\right)$. On the other hand, by MVT, $\dfrac{A_2 - A_1}{r_2 - r_1} = 2\pi r$ for some $r \in \left(r_1, r_2\right)$. Thus, $A_2 - A_1 = 2\pi r\left(r_2 - r_1\right)$. This problem is a fairly simple application of the MVT, and the fact that you can get the answer to be exactly the area of that rectangle will probably surprise students. This problem will be a good reference later on, when we are trying to prove the evaluation part of the FTC.

PTS: 1 DIF: Calculus 1 REF: 4.3
OBJ: Mean Value Theorem and shapes of curves KEY: rates of change
MSC: 248

115. ANS: D

This is a challenging problem for students. The main point here is to recall the idea (that was first introduced to them in the IVT) that we can show that two functions take the same value at a point by showing that their difference is zero. As the MVT only talks about what happens to one function, then we must look at the difference of the two functions in order to compare them at the same moment. Also discussing the problem with a graph, and showing that this happens when the slower person begins to speed up to catch up the other one will make it more clear.

PTS: 1 DIF: Calculus 1 REF: 4.3
OBJ: Mean Value Theorem and shapes of curves MSC: 415

116. ANS: C

The volume of the cone of radius r and height h is $V(h) = \frac{\pi}{3} r^2 h$. Looking at similar triangles, $r = h\dfrac{r_1}{h_1}$;

thus, $V(h) = \dfrac{\pi}{3} h^3 \left(\dfrac{r_1}{h_1}\right)^2$ and $V'(h) = \pi h^2 \left(\dfrac{r_1}{h_1}\right)^2 = \pi r^2$. Now using the MVT,

$$\frac{\Delta V}{\Delta h} = \frac{V(h_2) - V(h_1)}{h_2 - h_1} = V'(h_*) = \pi (r_*)^2$$

for some $h_* \in [h_1, h_2]$ and corresponding $r_* \in [r_1, r_2]$. Therefore, the volume of the slice is $\Delta V = \pi (r_*)^2 \Delta h$, equal to the volume of a cylinder of height Δh and radius r_* .

PTS: 1 DIF: Calculus 1 REF: 4.3
OBJ: Mean Value Theorem and shapes of curves MSC: 482

117. ANS: D

A repeated use of L' Hospital's Rule on their ratio gives the result.

PTS: 1 DIF: Calculus 1 REF: 4.5 OBJ: L' Hospital's Rule
MSC: 000

118. ANS: D

An application of L' Hospital to $\infty - \infty$ type of limits.

$$\lim_{x \to \infty} \left[x e^{1/x} - x \right] = \lim_{x \to \infty} \frac{e^{1/x} - 1}{1/x} = \lim_{x \to \infty} e^{1/x} = 1$$

PTS: 1 DIF: Calculus 1 REF: 4.5 OBJ: L' Hospital's Rule
KEY: l'hospital MSC: 552

119. ANS: D

Here the student can notice that the linear approximation to the functions is enough (when the slopes are both non-zero) to give their relative rate of change at a point. We have seen before the same problem, but with just the lines. This will give more geometric intuition as to why L' Hospital's Rule works.

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} = \frac{\textit{slope of } L_1}{\textit{slope of } L_2} = 2.$$

PTS: 1 DIF: Calculus 1 REF: 4.5 OBJ: L' Hospital's Rule
MSC: 469

120. ANS: B

Students tend to use the second derivative test without thinking, but in finding absolute extrema on a closed interval, it is not necessary. This also reminds students to check the endpoints in optimization problems.

PTS: 1 DIF: Calculus 1 REF: 4.6 OBJ: Optimization
KEY: optimization MSC: 432

121. ANS: B

(c) omits the possibility that the max or min could occur at a point on the graph where $f'(x)$ does not exist (such as a cusp), and (b) is false by the Extreme Value Theorem.

PTS: 1 DIF: Calculus 1 REF: 4.6 OBJ: Optimization
MSC: 552

122. ANS: D

If r is the radius of the circle, and x is the side of the square, $A = \pi r^2 + x^2$, and $L = 2\pi r + 4r$. After writing A as a function of r (or x), differentiating, testing at critical points and endpoints, one can easily see that we get a maximal enclosed area for $x = 0$ and $r = \dfrac{L}{2\pi}$. One can also reason geometrically, and students should be encouraged to use their intuition and think of a geometric argument before starting the analytic solution.

PTS: 1 DIF: Calculus 1 REF: 4.6 OBJ: Optimization
MSC: 448

123. ANS: B

A quick check see how Newton's method works graphically.

PTS: 1 DIF: Calculus 1 REF: 4.8 OBJ: Newton's Method
KEY: Newton's Method MSC: 542

124. ANS: D

Being lucky means that c is already the root of the function. A graphical explanation would make it very clear.

PTS: 1 DIF: Calculus 1 REF: 4.8 OBJ: Newton's Method
MSC: 612

125. ANS: A

Quick check of when we use Newton's method.

PTS: 1 DIF: Calculus 1 REF: 4.8 OBJ: Newton's Method
MSC: 560

126. ANS: B

This follows from the definition of antiderivative. This kind of problem makes the connection between antiderivatives and derivatives.

PTS: 1 DIF: Calculus 1 REF: 4.9 OBJ: Antiderivatives
KEY: antiderivatives MSC: 560

127. ANS: D

Students often hold onto the idea that the midpoint estimate is the best. With this example the instructor can point out that there are cases in which it does not work as well as some of the other height choices. A specific function which has a long thin spike at the midpoint is a good counterexample.

PTS: 1 DIF: Calculus 1 REF: 5.1 OBJ: Areas and Distances
MSC: 409

128. ANS: A
This is a quick application of the area approximations in this section. Once students understand the set up of the problem, the right answer should follow immediately from the text reading.

PTS: 1 DIF: Calculus 1 REF: 5.1 OBJ: Areas and Distances
KEY: areas, distances MSC: 542

129. ANS: B
This problem attempts to clarify most of the misconceptions that students have about definite integrals, and to help them move away from the idea that they always represent the area under a curve.

PTS: 1 DIF: Calculus 1 REF: 5.2 OBJ: The Definite Integral
KEY: definite integral MSC: 469

130. ANS: D
This question might also help students see the definite integral as total change rather than the standard area interpretation. At the same time it differentiates it from averages.

PTS: 1 DIF: Calculus 1 REF: 5.2 OBJ: The Definite Integral
MSC: 432

131. ANS: D
Recall the pizza problem (#4 in Section 3.1, #4 in Section 4.2). Students should easily see that (*b*) and (*c*) are equivalent. However, $\lim_{n \to \infty} \mathrm{A}_n = \mathrm{A}_n$ for all *n*. There is a point to be made about smarter and more efficient ways to estimate integrals, other than doing it with rectangles. The instructor should demonstrate to the students that the sum of the areas of sectors is the area of the whole disk.

PTS: 1 DIF: Calculus 1 REF: 5.2 OBJ: The Definite Integral
MSC: 541

132. ANS: B
See explanation for previous problem. Students should note the difference in expressions for A_n and integrals in the 2 problems above.

PTS: 1 DIF: Calculus 1 REF: 5.2 OBJ: The Definite Integral
MSC: 409

133. ANS: C
A point should be made, that this ’standard’ way of finding integrals, is actually much harder to calculate as is requires a special kind of substitution.

PTS: 1 DIF: Calculus 1 REF: 5.2 OBJ: The Definite Integral

134. ANS: A
Another quick check of what the definite integral measures. Instructors should differentiate between this integral and $\int_0^{60} v(t)dt$, in order to emphasize the difference between displacement and distance traveled.

PTS: 1 DIF: Calculus 1 REF: 5.3 OBJ: Evaluating Definite Integrals
KEY: definite integral, evaluating MSC: 542

135. ANS: B
This problem is to test whether students understand the diference between definite and indefinite integrals. The instructor should bring to their attention, that the answer they would write down in case $f(0) = 0$ would be the same.

PTS: 1 DIF: Calculus 1 REF: 5.3 OBJ: Evaluating Definite Integrals
MSC: 612

136. ANS: C
f is the derivative of G, thus $f > 0$ implies $G' > 0$, and therefore G is increasing. This is to demonstrate to students that they can apply the Increasing/Decreasing Test outside the context of problems like those in Chapter 4. Note that here we are just referring to the antiderivative of f. (a) may be a popular answer since we think of an integral of a positive function as ”adding” positive small pieces. But the choice between (a)or (b) for a particular antiderivative G, depends on the constant.

PTS: 1 DIF: Calculus 1 REF: 5.3 OBJ: Evaluating Definite Integrals
MSC: 560

137. ANS: A
Follows directly from properties of the integral.

PTS: 1 DIF: Calculus 1 REF: 5.4
OBJ: The Fundamental Theorem of Calculus KEY: FTC, Fundamental Theorem
MSC: 560

138. ANS: D
This problem can help students realize that an integral of this form is a specific function (not a general antiderivative), and the integrand is its derivative. Students can induce information about the concavity of g just like they normally would with any other function, only that this time they have to look at f.

PTS: 1 DIF: Calculus 1 REF: 5.4
OBJ: The Fundamental Theorem of Calculus MSC: 494

139. ANS: C
$g(0) = 0$, $g'(0) = 4$ and $g'(2) = f(2) = 0$

PTS: 1 DIF: Calculus 1 REF: 5.4
OBJ: The Fundamental Theorem of Calculus MSC: 494

140. ANS: A
This is a great problem to put together the standard definition of the average velocity, the MVT and also the representation of total distance traveled from integration. It also introduces them to the MVT for integrals.

PTS: 1 DIF: Calculus 1 REF: 5.4
OBJ: The Fundamental Theorem of Calculus MSC: 542

141. ANS: A
It will probably help students to remember where the Substitution rule comes from, instead of simply memorizing a rule.

PTS: 1 DIF: Calculus 1 REF: 5.5 OBJ: The Substitution Rule
KEY: substitution rule MSC: 415

142. ANS: D
This is an immediate consequence of the substitution rule. At time $t = 0, A = \left(g(0)\right)^2$ and at $t = 1$, $A = \left(g(1)\right)^2$. So (a) follows. We get (b) from $dA = f(r)dr$, and (c) from $dA = f(r)dr = f\left(g(t)\right)g'(t)dt$.

PTS: 1 DIF: Calculus 1 REF: 5.5 OBJ: The Substitution Rule
MSC: 482

143. ANS: C
This is an application of the previous problem. $A(t) = \pi r^2 = \pi(\ln(t+2))^2$ at $t = 0$, $r(t) = \ln 2$, and at $t = 1$, $r(t) = \ln 3$. Then (a) clearly follows. (b) holds since $A'(r) = 2\pi r$, and thus $\Delta A = \int_{\ln 2}^{\ln 3} A'(r)dr$. (c) follows from $\Delta A = \int_0^1 A'(t)dt$.

PTS: 1 DIF: Calculus 1 REF: 5.5 OBJ: The Substitution Rule
MSC: 000

144. ANS: C
If we let $y = \sin t$, and $x = \cos t$, then $dx = -\sin t\, dt$. When $x = -1$, $t = \pi$ and $x = 1$, $t = 0$; therefore, $A = \int_\pi^0 -\sin^2 t dt$.

PTS: 1 DIF: Calculus 1 REF: 5.5 OBJ: The Substitution Rule
MSC: 552

PROBLEM

145. ANS:
The activity is designed to get the students to practice with the idea of estimating, as well as setting up a Riemann Sum, starting with tangible examples.

PTS: 1 DIF: Calculus 1 REF: 5.1 OBJ: Areas and Distances
MSC: 415

146. ANS:
Triangle. The volume of the whole carrot is $V = \dfrac{B \cdot H}{3}$, where $B = r^2 \pi$.

PTS: 1 DIF: Calculus 1 REF: 5.1 OBJ: Areas and Distances
MSC: 482